Microeconomics III - Assignment 3

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Problem 1

a)

We know that the valuation for player i is:

$$v_i = \theta_1 + \theta_2$$

We formulate player i's expected payoff when player j bids $b_j = \theta_j$

$$E(u_i|b_i) = P(\text{i wins}|b_i) * (payoff|\text{i wins})$$

We first find:

$$(payoff|\text{i wins}) = \theta_i + E(\theta_i) - b_i$$

$$E(\theta_j) = \frac{b_i}{2}$$

We know that: $\theta_i = u[0, 100]$ The expected payoff becomes:

$$Pr(b_i \geq \theta_j) \cdot E(\theta_i + \theta_j - b_i)$$

$$Pr(b_i \geq \theta_j) \cdot (\theta_i + \frac{b_i}{2} - b_i)$$

$$Pr(b_i \geq \theta_j) \cdot (\theta_i - \frac{b_i}{2})$$

$$CDF : F(x) = \frac{x - a}{b - a} \Rightarrow P(k > x) = \frac{k - a}{b - a}$$

Using CDF we get:

$$\frac{b_i}{100} \cdot (\theta_i - \frac{b_i}{2})$$

$$BR_i = \frac{b_i\theta_i}{100} - \frac{b_i^2}{100 \cdot 2}$$

Taking the FOC wrt. b_i

$$\frac{\partial BR_i}{\partial b_i} = \frac{\theta_i}{100} - \frac{b_i}{100} = 0$$

$$\theta_i = b_i$$

b)

We know that the players only get the object if their bid is winning. Therefore, they should would only care about the expected value of the object conditional on their bid winning. We can calculate, in the equilibrium, the expected value of the object conditional on winning:

$$\theta_i + E(\theta_j | b_i > b_j) = \theta_i + \frac{b_i}{2}$$

If we have the $\theta_i = 1$, we have the expected value conditioned on winning being 1, while the actual expected value is 51.

c)

We now solve the BNE where players use linear strategies: We know that $v_i = 2\theta_i$, since player i doesn't know the true value of player j. The same is true for the opposite. We still have that $\theta_i \sim uw[0, 100]$

We formulate player i's expected payoff when player j bids $b_j = \theta_j$

$$E(u_i|b_i) = P(\text{i wins}|b_i) * (payoff|\text{i wins})$$

We first find:

$$(payoff|\text{i wins}) = 2\theta_i + -b_i$$

Player i doesn't know the valuation of player j, so they assume it's given by a linear combination of player j's type. This gives us $b_j(v_j) = \beta\theta_j$. The expected payoff becomes:

$$Pr(b_i \geq b_j(v_j)) \cdot E(2\theta_i - b_i) = Pr(b_i \geq \beta\theta_j) \cdot (2\theta_i - b_i)$$

$$= Pr(\frac{b_i}{\beta} \geq \theta_j) \cdot (2\theta_i - b_i) = \frac{\frac{b_i}{\beta}}{100} \cdot (2\theta_i - b_i)$$

$$CDF : F(x) = \frac{x-a}{b-a} \Rightarrow P(k > x) = \frac{k-a}{b-a}$$

Using CDF we get:

$$BR_i = \frac{b_i}{100 \cdot \beta} \cdot (2\theta_i - b_i) = \frac{b_i 2\theta_i}{100 \cdot \beta} \cdot - \frac{b_i^2}{100 \cdot \beta}$$

Taking the FOC wrt. b_i

$$\frac{\partial BR_i}{\partial b_i} = \frac{2\theta_i}{100 \cdot \beta} - \frac{2b_i}{100 \cdot \beta} = 0$$

$$\Leftrightarrow \frac{\theta_i}{100 \cdot \beta} - \frac{b_i}{100 \cdot \beta} = 0 \Leftrightarrow \theta_i = b_i$$

The Nash Equilibrium is then given by $b_i^*(\theta_i) = \theta_i$ for $i = 1, 2$.

d)

We end up showing that the players will bid $b_i = \theta_i$ for $i = 1, 2$ in common and private auctions. In the common auction, the payoff was given by $v_i = \theta_i + \theta_j - \beta_i$ while it was given as $v_i = 2\theta_i - \beta_i$ in the private auction. As long as $\theta_i > \theta_j$, player i will win and get more value in the private auction. The players will always gain more value in the private first price auction compared to the common first price auction. This is known as the Winner's Curse, which states that the expected value of a common object changes conditional on winning the auction. The players will simply pay too much for an item.

Problem 2

Signalling game steps
1. R: Find the beliefs p, q given S's eq strategy.
2. R: Given beliefs find BR_R
3. S: does t_1 or t_2 want deviate.
We have following strategy set:

$$S_s = (L, L)(R, R)(L, R)(R, L)$$

1. We start solving for following strategy: (L,L)

$$\mu(t_1|L) = p = \frac{1}{2}$$

$$\mu(t_1|R) = q \in [0,1] \text{ no beliefs}$$

2. Finding BR for playing L

$$E[u_R(L,u)] = 2p + 1(1-p) = \frac{3}{2}$$

$$E[u_R(L,d)] = 1p + 3(1-p) = 1p + 3 - 3p = \frac{1}{2} + \frac{6}{2} - \frac{3}{2} = 2$$

$$BR_R(L) = d$$

3.
$s_1^{t_1}$ will not deviate if d is played given R.
$s_1^{t_2}$ will not deviate if d is played given R.
We calculate the expected payoff for player 2 if R is played in order to find the value of q. We do this in order to find a BR off-path.

$$E[u_R(R,u)] \geq E[u_R(R,d)]$$
$$2q + 3(1-q) \geq 1q + 1(1-q) \Leftrightarrow$$
$$1 \geq 3 - q \Leftrightarrow q \geq 2$$

As long as $q \geq 2$ then u will be a BR, otherwise, d is a BR. We find that (L,L) is a PBE:

$$PBE = \{(L,L),(d,d), p = 1/2, q \geq 2)\}$$

1. Now we solve for this strategy set: (R,R)

$$\mu(t_1|R) = q = 1/2$$

$$\mu(t_1|R) = p \in [0,1] \text{ no beliefs}$$

2. BR

$$E[u_R(R,u)] = 2q + 3(1-q) = \frac{5}{2}$$

$$E[u_R(R,d)] = 1q + 1(1-q) = 1$$

$BR_R(R) = u$
3.

t_1 will not deviate. t_2 will not deviate.

$$E[u_R(L,u)] \geq E[u_R(L,d)]$$

$$2p + 1(1-p) \geq 1p + 3(1-p) \Leftrightarrow$$

$$p + 1 \geq 3 - 2p) \Leftrightarrow 3p \geq 2 \Leftrightarrow p \geq \frac{2}{3}$$

We find the a PBE:

$$PBE = \{(R,R),(d,d), q = 1/2, p \geq \frac{2}{3})\}$$

1. Now we solve for this strategy (L,R)

$$s_1^{t_1} = L, s_1^{t_2} = R,$$

$$\mu(t_1|L) = p = 1$$

$$\mu(t_1|R) = q = 0$$

2. BR for playing L

$$E[u_R(L,u)] = 2$$

$$E[u_R(L,d)] = 1$$

$$BR_R(L) = u$$

BR for Playing R

$$E[u_R(R,u)] = 3$$

$$E[u_R(R,d)] = 1$$

$$BR_R(R) = u$$

3. If $s_1^{t_1}$ deviates from playing L to R we see that $s_1^{t_1}$ would not be better of playing R because the payoff will be the same.
If $s_1^{t_2}$ deviates from playing R to L, we see that $s_1^{t_2}$ would not be better off playing L because the payoff would then be smaller. Therefore we have a PBE:

$$PBE = \{(L,R)(u,u), p = 1, q = 0\}$$

1. For this strategy set we have: (R,L)

$$\mu(t_1|L) = p = 0$$

$$\mu(t_1|R) = q = 1$$

2. BR for Playing R

$$E[u_R(R,u)] = 2$$
$$E[u_R(R,d)] = 1$$
$$BR_R(R) = u$$

BR for playing L

$$E[u_R(L,u)] = 1$$
$$E[u_R(L,d)] = 3$$
$$BR_R(L) = d$$

3. If $s_1^{t_1}$ deviates from playing R to L we see that $s_1^{t_1}$ would be better of playing R because the payoff will be higher.

This means that there is no PBE.

$$PBE = \begin{cases} \{(R,R)(d,d), q = 1/2 & p \geq \frac{2}{3}\} \\ \{(L,L),(d,d), p = 1/2, q \geq 2)\} \end{cases}$$