Microeconomics III - Assignment 2

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1.

a)

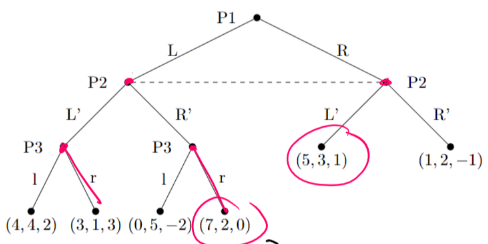
It is a game of imperfect information. 3 subgames if we include the game itself. Therefore 2 proper subgames.
The players has the following strategy sets:

$$\{S_1, S_2, S_3\} = \{(L, R), (L', R'), (ll, lr, rl, rr)\}$$

b)

We have 2 Subgame perfect equilibria.

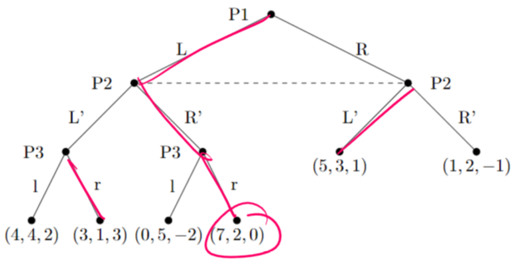
$$SPNE = \{(L), (R'), (rr)\}$$
$$SPNE = \{(R), (L'), (rr)\}$$



		P2: L'	P2: R'
P1	L	3, 1	7, 2
	R	5, 3	1, 2

c)

As player 2 can now observe player 1´s action, we only have one SPE now, because player 1 will also react to player 2´s new actions.



d)

Player 2 will lose on average compared to question a).

2.

a)

Consider the following normal form game:

$$\begin{array}{cccc} & a & b & c \\ A & (0,0) & (\underline{4},\underline{4}) & (\underline{6},0) \\ B & (\underline{1},\underline{2}) & (0,0) & (2,\underline{2}) \\ C & (0,\underline{6}) & (2,2) & (5,5) \end{array}$$

We find two pure NE by using best responses:

$$NE = \{(A,a)(B,b)\}$$

And the expected payoffs are:

$$EP = \{(1,2)(4,4)\}$$

The strategies are:
Player two plays the mixed strategy:

$$E(A) = 0q_1 + 4q_2 + 6(1 - q_1 - q_2) = 6 - 6q_1 - 2q_2$$

$$E(B) = 1q_1 + 0q_2 + 2(1 - q_1 - q_2) = 2 - 1q_1 - 2q_2$$

$$E(C) = 0q_1 + 2q_2 + 5(1 - q_1 - q_2) = 5 - 5q_1 - 3q_2$$

$$E(A) = E(B) \Leftrightarrow 6 - 6q_1 - 2q_2 = 2 - 1q_1 - 2q_2 \Leftrightarrow \frac{4}{5} = q_1$$

$$6 - 6q_1 - 2q_2 = 5 - 5q_1 - 3q_2 \Leftrightarrow 6 - 6 \cdot \frac{4}{5} - 2q_2 = 5 - 5 \cdot \frac{4}{5} - 3q_2$$

$$\Leftrightarrow q_2 = -\frac{1}{5}$$

Since player 2 cannot play q_2 with negative probability, we assume $q_2 = 0$. Then we get the probability of player 2 playing C to be:

$$C = 1 - q_1 - q_2 = 1 - \frac{4}{5} - 0 = \frac{1}{5}$$

For player 1, we get the strategies:

$$E(a) = 0p_1 + 2p_2 + 6(1 - p_1 - p_2) = 6 - 6p_1 - 4p_2$$

$$E(b) = 4p_1 + 0p_2 + 2(1 - p_1 - p_2) = 2p_1 + 2 - 2p_2$$

$$E(c) = 0p_1 + 2p_2 + 5(1 - p_1 - p_2) = 5 - 5p_1 - 3p_2$$

$$6 - 6p_1 - 4p_2 = 5 - 5p_1 - 3p_2 \Leftrightarrow p_1 = -p_2 + 1$$

$$2(-p_2 + 1) + 2 - 2p_2 = 5 - 5(-p_2 + 1) - 3p_2 \Leftrightarrow p_2 = \frac{2}{3}$$

$$p_1 = -\frac{2}{3} + 1 = \frac{1}{3}$$

The final mixed strategies are:

$$Player1 : \{(p_1, p_2, 1 - p_1 - p_2)\} = \{(\frac{1}{3}, \frac{2}{3}, 0)\}$$

$$Player2 : \{(q_1, q_2, 1 - q_1 - q_2)\} = \{(\frac{4}{5}, 0, \frac{1}{5})\}$$

The expected payoff in the mixed strategy becomes:

$$EP(Player1) = \frac{1}{3}\left(\frac{4}{5} \cdot 0 + \frac{1}{5} \cdot 6\right) + \frac{2}{3} \cdot \left(\frac{4}{5} \cdot 1 + \frac{1}{5} \cdot 2\right) = \frac{6}{5}$$

$$EP(Player2) = \frac{4}{5}\left(\frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 2\right) + \frac{1}{5}\cdot\left(\frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 2\right) = \frac{4}{3}$$

b)

This would still be a game of imperfect information as the players still have to make a simultaneous choice in the first round, where they don't know the final choices. Furthermore they also don't know the others choice in round 2.

c)

If both players play the SPE strategy (A,b) as long as they play (C,c) in round 1, and they play (B,a) as long as they don't play (C,c) in round 1, we get the following:

	a	b	c
A	0+1,0+2	4+1,4+2	6+1,0+2
B	1+1,2+2	0+1,0+2	2+1,2+2
C	0+1,6+2	2+1,2+2	5+4,5+4

We now find that playing (C,c) in the first round gives the expected payoff of 9 to both players. This result is much higher than the payoffs in question a. When playing two rounds, the highest payoffs they can get while playing pure NE is (8,8) while playing (Ab,Ab).

3.

a)

We construct a trigger strategy profile where (C,c) is always played on the equilibrium path:

$$U_1(C,c) = \sum_{t=1}^{\infty} \delta_1^{t-1} \cdot 5 = \frac{5}{1-\delta_1}$$

$$U_1(B,a) = \sum_{t=1}^{\infty} \delta_1^{t-1} \cdot 1 = \frac{1}{1-\delta_1}$$

$$U_1(C,c) \geq U_1(B,a) \Leftrightarrow \frac{5}{1-\delta_1} \geq \frac{1}{1-\delta_1} \Leftrightarrow 5 \geq 1$$

In this strategy, the players play (B,a), as long as a player no longer plays (C,c). We find that playing (C,c) is always better on path.

b)

We now find the smallest discount factor such that the profile gives a SPE:

$$U_1(C,c) = \sum_{t=1}^{\infty} \delta_1^{t-1} \cdot 5 = \frac{5}{1-\delta_1}$$

$$U_1(A,c) = 6 + \sum_{t=2}^{\infty} \delta_1^{t-1} \cdot 1 = 6 + \frac{\delta_1}{1-\delta_1}$$

$$U_1(C,c) = U_1(A,c) \Leftrightarrow \frac{5}{1-\delta_1} = 6 + \frac{\delta_1}{1-\delta_1} \Leftrightarrow$$

$$5 = 6(1-\delta_1) + \delta_1 \Leftrightarrow \delta_1 = \frac{1}{5}$$

$$U_2(C,c) = \sum_{t=1}^{\infty} \delta_2^{t-1} \cdot 5 = \frac{5}{1-\delta_2}$$

$$U_2(A,c) = 6 + \sum_{t=2}^{\infty} \delta_2^{t-1} \cdot 2 = 6 + \frac{2\delta_2}{1-\delta_2}$$

$$U_1(C,c) = U_1(A,c) \Leftrightarrow \frac{5}{1-\delta_2} = 6 + \frac{2\delta_2}{1-\delta_2} \Leftrightarrow$$

$$5 = 6(1-\delta_2) + 2\delta_2 \Leftrightarrow \delta = \frac{1}{4}$$