UNIVERSITY OF COPENHAGEN

Mikro II, lecture 9b

Cournot with many firms

Johannes Wohlfart

Plan for the lecture

1. Comparison between complete competition, monopoly and Cournot
2. The Cournot model with N companies

Setup

- We consider a market with a linear demand curve $p(x) = a - bx$ and N firms
- Each firm has the cost function $C(x_i) = K\frac{1}{2}x_i^2$, and the total quantity in the market is $x = \sum_i x_i$
- A little strange assumption (but will be useful later):
  each company can produce a maximum of $\frac{1}{K}$ units (i.e. $0 \leq x_i \leq \frac{1}{K}$), but the demand is such that this restriction *never* binds
- Interpretation of K (productivity): A lower K means that each company can produce a larger quantity at the same price (and vice versa)

Monopoly

- Suppose $N = K = 1$ and the company behaves as a monopolist
- Usual $MR = MC$ solution provides (check yourself): $x^m = \frac{a}{2b+1}$

Perfect competition

- Now assume $N \geq 1$ but again $K = N$ and that companies behave like price takers
- Usual $p = MC$ solution provides supply curve for company i (check yourself): $x_i^*(p) = \frac{p}{K}$
- Total supply curve $x^*(p) = \sum_i x_i^*(p) = \frac{N}{K}p = p$
- Usual equilibrium condition yields the equilibrium quantity: $x^f = \frac{a}{b+1}$

Perfect competition, discussion

- Note that the total supply curve, $x^*(p)$ does not depend on the number of firms N and so doesn't the equilibrium quantity

- Intuition:
  - We have set $K = N$ ie. when we change the model to have more firms, we at the same time make them less productive

  - A high N therefore means many low-productivity firms with low output, while low N means few high-productivity firms with high output.

- Note that the model here formally demonstrates that perfect competition with a single company can serve as approximation to many firms (representative firm)

Cournot oligopoly

- We now continue to look at $N \geq 1$ and $K = N$, but now assume Cournot competition

- Firm i maximizes given the others' decisions:

$$\max_{x_i} \quad p\left(\sum_j x_j\right) x_i - C(x_i)$$

- FOC:

$$p\left(\sum_j x_j\right) + p'\left(\sum_j x_j\right) x_i - C'(x_i) = 0$$

Solution

- Insert functional form (and $\sum_j x_j = x$) :

$$a - bx - bx_i - Kx_i = 0 \iff$$

$$x_i = \frac{a - bx}{b + K}$$

- In a Nash equilibrium this equation must be fulfilled for all i, it follows that the equilibrium must be symmetric: $\bar{x}_i = \bar{x}$

- Insert in the first-order condition to find the equilibrium (implicitly we find the best response):

$$a - bN\bar{x} - b\bar{x} - K\bar{x} = 0 \iff$$

$$\bar{x} = \frac{a}{(N+1)b + K}$$

Cournot eqilibrium

- Nash equilibrium under Cournot oligopoly $\bar{x}_i = \frac{a}{(N+1)b+N}$ for all i (remember $N = K$)

- The total quantity will then be:

$$x^c = \sum_i \bar{x}_i = N\bar{x}_i = N\frac{a}{(N+1)b+N} = \frac{a}{\frac{(N+1)}{N}b+1}$$

- Easy to check that total quantity is increasing in N here and thus the price is falling; more companies => more competition

Cournot with different N

$$x^c = \frac{a}{\frac{(N+1)}{N}b + 1}$$

- If $N = 1$, we get the same equilibrium total quantity (and price) as under monopoly:

$$x^c = \frac{a}{2b+1} = x^m$$

- If $N \to \infty$ (that is N very big) we have the same quantity (and price) as under perfect competition (remember $\lim_{N\to\infty} \frac{(N+1)}{N} = 1$):

$$\lim_{N\to\infty} x^c = \frac{a}{b+1} = x^f$$

Discussion

- So far we have argued that perfect competition (and price-taking) is a good approximation when there are many small businesses

- The Cournot model we just reviewed formalizes this idea mathematically:
  - Monopoly is a special case that occurs when $N = 1$ (and $K = N$) i.e. there is a single company (with large production)

  - Perfect competition is a special case that occurs when $N \longrightarrow \infty$ (and $K = N$) ie. there are a lot of companies (with little production)

Socrative Quiz Question

True or false: So far we have assumed that all firms have the same cost function. Imagine that the setup is the same as previously, but one company has access to a better production technology allowing cheaper production. In the new equilibrium, this firm will take the entire market, and production of all the other firms will be zero.

A little more on $N \longrightarrow \infty$

- Obvious why $N = 1$ equals monopoly (maximization problem identical between monopoly and Cournot)

- Let's take a closer look at what happens when $N \longrightarrow \infty$

- Here it becomes useful to re-parameterize the problem / decision of companies:
  - The companies produce a percentage $\frac{1}{K}$ of maximum production, i.e. they choose s_i, where $x_i = \frac{1}{K} s_i$

Re-parameterized problem

- Old problem:

$$\max_{x_i} \quad p\left(\sum_j x_j\right) x_i - C(x_i)$$

- Re-parameterized problem:

$$\max_{s_i} \quad p\left(\sum_j \tfrac{1}{K} s_j\right) \tfrac{1}{K} s_i - C(\tfrac{1}{K} s_i)$$

- Because there is a 1-to-1 relationship between s_i and x_i, these problems are mathematically equivalent

First-order condition in the new problem I

$$\max_{s_i} \quad p\left(\sum_j \frac{1}{K}s_j\right)\frac{1}{K}s_i - C\left(\frac{1}{K}s_i\right)$$

- The first order condition only holds for marginal changes of s_i:

$$\underbrace{p\left(\sum_j \frac{1}{K}s_j\right)\frac{1}{K}}_{\text{Revenue from another } \frac{1}{K} \text{ units}} + \underbrace{p'\left(\sum_j \frac{1}{K}s_j\right)\frac{1}{K}\cdot\frac{1}{K}s_i}_{\text{Effect on price of other units}} = \underbrace{C'\left(\frac{1}{K}s_i\right)\frac{1}{K}}_{\text{Marginal cost of another } \frac{1}{K} \text{ units}}$$

First-order condition in the new problem II

- Insert x to make it look nicer and insert from the cost function:

$$\underbrace{p(x)\frac{1}{K}}_{\text{Revenue from an additional } \frac{1}{K} \text{ units}} + \underbrace{p'(x)\frac{1}{K}\cdot\frac{1}{K}s_i}_{\text{Effect on other units' price}} = \underbrace{s_i\frac{1}{K}}_{\text{Marginal cost of } \frac{1}{K} \text{ units}}$$

- Multiply with K (scales to "per unit" instead of "per $\frac{1}{K}$ units"):

$$\underbrace{p(x)}_{\text{Revenue from an additional unit}} + \underbrace{p'(x)\frac{1}{K}s_i}_{\text{Effect on other units' price}} = \underbrace{s_i}_{\text{Marginal cost}}$$

First-order condition in the new problem III

- Then insert our assumption of $N = K$ and evaluate in perfectly competitive equilibrium (x^c)

$$\underbrace{p\left(x^c\right)}_{\text{Revenue from an additional unit}} + \underbrace{p'\left(x^c\right)\frac{1}{N}s_i}_{\text{Effect on other units' price}} = \underbrace{s_i}_{\text{Marginal cost}}$$

- Now let $N \rightarrow \infty$ for a given s_i (and remember that we know that the equilibrium quantity goes toward x^f):

$$\underbrace{p\left(x^f\right)}_{\text{Revenue from an additional unit}} + \underbrace{0}_{\text{Effect on other units' price}} = \underbrace{s_i}_{\text{Marginal cost}}$$

First-order condition in the new problem IV

$$\underbrace{p\left(x^f\right)}_{\text{Revenue from an additional unit}} + \underbrace{0}_{\text{Effect on other units' price}} = \underbrace{s_i}_{\text{Marginal cost}}$$

- We see that when N becomes very large, the effect of own production on the price disappears from the first-order condition; means the company becomes a price-taker! (and solution becomes p = MC)

- (Why did we introduce s_i? To avoid looking at a zero-solution maximization problem: Note that $\lim_{N\to\infty} x_i = 0$, while $\lim_{N\to\infty} s_i = \frac{a}{b+1} > 0$)

What have we learned?

- To solve the Cournot model with N companies
- The connection between monopoly, Cournot and perfect competition