Mikro II, lecture 9a

Oligopoly, Cournot and Bertrand

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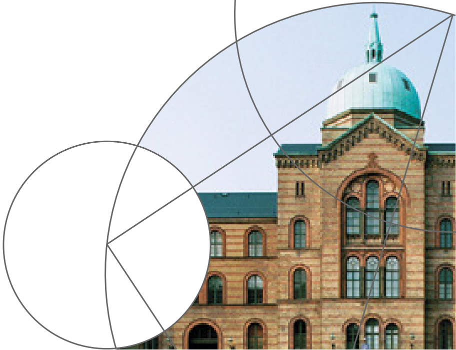

Plan for the lecture

1. Cournot's model of oligopoly with two firms
2. Bertrand's model of oligopoly with two firms
3. Comparison of the two models
4. Extensions to non-sequential games

Setup

- A market with a linear demand curve $p(x) = a - bx$.
- Production takes places at constant marginal cost $C(x) = c \cdot x$
- Recall our previous results (with $a > c$):
  - Efficient quantity given by $x^e = \frac{a-c}{b}$
  - With perfect competition (price-taking firms) we get $x^f = \frac{a-c}{b}$ and $p^f = c$
  - With a monopoly (one price-setting firm) we get $x^m = \frac{1}{2}\frac{a-c}{b}$ and $p^m = \frac{a+c}{2}$

Technology and preferences

Technology and Preferences	**Behavior and Equilibrium**
Exogenous functions / var. / relationships: $p(x) = a - b \cdot x$ $C(x) = c \cdot x$ a, b, c *Endogenous variables:* p, x	*The decisions of the agents:* $\hookrightarrow$ *Conditional behavior:* *Equilibrium Conditions:*

Perfect competition

Technology and Preferences	Behavior and Equilibrium
Exogenous functions / var. / relationships:	*The decisions of the agents:*
$p(x) = a - b \cdot x$	Price-taking of firms
$C(x) = c \cdot x$	$\max_x p \cdot x - C(x)$
a, b, c	
	$\hookrightarrow$ *Conditional behavior:*
Endogenous variables:	Perfectly elastic output
p, x	with price c
	Equilibrium Conditions:
	$p^f = c, \quad p(x^f) = p^f$

Monopoly

Technology and Preferences	Behavior and Equilibrium
Exogenous functions / var. / relationships: $p(x) = a - bx$ $C(x) = c \cdot x$ a, b, c *Endogenous variables:* p, x	*The decisions of the agents:* Price-setting of monopolist $\max_x p(x) \cdot x - C(x)$ $\hookrightarrow$ *Conditional behavior:* x^m *Equilibrium Conditions:* $p^m = p(x^m)$

Oligopoly

- We keep our assumptions on technology and preferences but change our assumption on the behavior / number of firms.
  - Compared to monopoly, we now have two similar firms producing x_1 and x_2, so $x = x_1 + x_2$.
  - Compared to perfect competition, we move away from the assumption of price-taking; the firms know that their production decisions will affect the price.

The two firms play an economic game

- The profits of the two firms now each depend on both x_1 and x_2:

$$\pi_1(x_1,x_2) = p(x_1+x_2)x_1 - C(x_1)$$
$$\pi_2(x_1,x_2) = p(x_1+x_2)x_2 - C(x_2)$$

- We can find the Nash equilibrium of this game.
  - The strategies are x_1 and x_2, so $x = x_1 + x_2$; the strategy space is $S_1 = S_2 = \mathcal{R}^+$
  - The "utility functions" are the firms' profit functions:

$$u_1(x_1,x_2) = \pi_1(x_1,x_2)$$
$$u_2(x_1,x_2) = \pi_2(x_1,x_2)$$

Oligopoly (Cournot)

Technology and Preferences	**Behavior and Equilibrium**
Exogenous functions / var. / relationships:	*The decisions of the agents:*
$p(x) = a - bx$	Profit max. given other's prod.
$C(x) = c \cdot x$	$\max_{x_1} p(x_1 + x_2) \cdot x_1 - C(x_1)$
a, b, c	$\max_{x_2} p(x_1 + x_2) \cdot x_2 - C(x_2)$
	$\hookrightarrow$ *Conditional behavior:*
Endogenous variables:	Best response
p, x	$x_1^*(x_2), x_2^*(x_1)$
	Equilibrium Conditions:
	Nash equilibrium
	$\bar{x}_1 = x_1^*(\bar{x}_2), \quad \bar{x}_2 = x_2^*(\bar{x}_1)$

Best response

- Firm 1's best response maximizes:

$$\pi_1(x_1, x_2) = p(x_1 + x_2)x_1 - C(x_1)$$

- First order condition:

$$p(x_1 + x_2) + p'(x_1 + x_2)x_1 = C'(x_1) \Leftrightarrow x_1 = \frac{a-c}{2b} - \frac{1}{2}x_2$$

- The firms are identical, so we get the exact same for firm 2:

$$x_2 = \frac{a-c}{2b} - \frac{1}{2}x_1$$

Socrative Quiz Question

True or false: There is no weakly dominant strategy for any of the two players in the Cournot game.

Nash equilibrium (I)

$$x_1^*(x_2) = \frac{a-c}{2b} - \frac{1}{2}x_2 \qquad x_2^*(x_1) = \frac{a-c}{2b} - \frac{1}{2}x_1$$

- Insert one best response into the other in order to find the equilibrium quantities for both firms:

$$\bar{x}_1 = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{2b} - \frac{1}{2}x_1\right) \Leftrightarrow \bar{x}_1 = \frac{a-c}{3b}$$

- Substituting into the other equation gives:

$$\bar{x}_2 = \frac{a-c}{3b}$$

Nash equilibrium (II)

- In equilibrium the total quantity offered and the price are:

$$\bar{x} = \bar{x}_1 + \bar{x}_2 = \frac{2}{3}\frac{a-c}{b} \qquad \bar{p} = p\left(\frac{2}{3}\frac{a-c}{b}\right) = \frac{a+2c}{3}$$

- The equilibrium quantity is between monopoly and perfect competition:

$$\underbrace{\frac{1}{2}\frac{a-c}{b}}_{\text{Monopoly}} < \frac{2}{3}\frac{a-c}{b} < \underbrace{\frac{a-c}{b}}_{\text{Perfect competition}}$$

- The equilibrium price is between monopoly and perfect competition:

$$\underbrace{\frac{a+c}{2}}_{\text{Monopoly}} > \frac{a+2c}{3} > \underbrace{c}_{\text{Perfect competition}}$$

Discussion

- In this model, the oligopoly equilibrium is between monopoly and perfect competition.
- The firms compete so much that the price is lower and the quantity is higher than under monopoly . . .
- . . . but not enough to achieve the efficient equilibrium as under perfect competition.
- Deadweight loss less than under monopoly, but still existent.

Socrative Quiz Question

Recall the games we discussed on set of slides 8b. Which game is the Cournot model closest to in spirit?

- a) Rock-paper-scissor
- b) Car game
- c) Battle of the sexes
- d) Prisoner's dilemma
- e) Nørrebrogade meet-up game

Prices vs quantities revisited

- We have now analyzed the first model of oligopoly: the *Cournot* model
- As in the case of perfect competition and (most of the time) monopoly, we have analyzed this assuming firms which choose quantities, not prices.
- In the case of monopoly, we have seen that it does not really matter whether we assume that the firm chooses price or quantity; this is not true in the case of oligopoly.
- We will now analyze another oligopoly model: the *Bertrand* model, where firms choose prices.

Oligopoly with choice of prices

- We stick to our previous assumptions on technology and preferences $(p(x) = a - bx,\ C(x) = c \cdot x,\ a > c)$
- We now explicitly use the demand function (and not only the inverse demand function): $D(p) = p^{-1}(p)$
- We now assume that each of the two firms selects a price p_1, p_2. We also make the following assumptions:
  - If the price of one firms is lower than the other firm's price, all of the consumers will buy from the firm with the lower price.
  - If the firms set the same price, they will split the consumers evenly between them.
  - The produced quantity follows from the demand function.

Revenue and profit

- Firm 1's sold quantity depends on prices as follows:

$$D_1^B(p_1,p_2) = \begin{cases} D(p_1) & \text{for} \quad p_1 < p_2 \\ \frac{D(p_1)}{2} & \text{for} \quad p_1 = p_2 \\ 0 & \text{for} \quad p_1 > p_2 \end{cases}$$

- Firm 1 maximizes profit:

$$\pi_1(p_1,p_2) = p_1 \cdot D_1^B(p_1,p_2) - c \cdot D_1^B(p_1,p_2) = (p_1 - c) \cdot D_1^B(p_1,p_2)$$

- Firm 2 is identical. Again, this is an economic game and we will look for the Nash Equilibrium.

Oligopoly (Bertrand)

Technology and Preferences	**Behavior and Equilibrium**
Exogenous functions / var. / relationships:	*The decisions of the agents:*
$p(x) = a - bx$	Profit max. given other's prod.
$C(x) = c \cdot x$	$\max_{p_1} (p_1 - c) \cdot D_1^B(p_1, p_2)$
a, b, c	$\max_{p_2} (p_2 - c) \cdot D_2^B(p_1, p_2)$
$D(p) = p^{-1}(p)$	
$D_1^B(p_1, p_2),\ D_2^B(p_1, p_2)$	$\hookrightarrow$ *Conditional behavior:*
Endogenous variables:	Best response $p_1^*(p_2),\ p_2^*(p_1)$
	Equilibrium Conditions:
x, p_1, p_2	Nash equilibrium
	$\bar{p}_1 = p_1^*(\bar{p}_2), \quad \bar{p}_2 = p_2^*(\bar{p}_1)$

Equilibrium analysis

- Note that the profit maximization problem in this model is special:
  - Choice variable p_1 is continuous.
  - Profit function is discontinuous in the opponent's price.
- Standard approach of finding best responses through first order conditions does not work.
- Instead, we find the best responses formally in three steps.

Step 1: $p_1, p_2 \geq c$ in equilibrium

1. Assume $p_2 < c$ in equilibrium
2. With $p_1 \leq p_2$ we get $D_1^B(p_1, p_2) > 0$ and $p_1 < c$, that means profits are strictly negative for 1.
   If instead $p_1 > p_2$ we get $D_1^B(p_1, p_2) = 0$ that is zero profit; thus $p_1 > p_2$
3. Since $p_1 > p_2$ and $D_2^B(p_2, p_1) > 0$ we get strictly negative profits for 2.
   if instead $p_2 > p_1$ we get $D_2^B(p_2, p_1) = 0$, that means zero profit so p_2 is not a best response $\Rightarrow$ no equilibrium
4. From the above contradiction it follows that every equilibrium must have $p_2 \geq c$;
   by the same argument it must hold in equilibrium that $p_1 \geq c$.

Step 2a: With $p_2 > c$ it holds in equilibrium that $p_1 < p_2$

1. Assume $p_2 > c$ in equilibrium and examine the decision of 1:
   - $p_1 > p_2$ gives zero profit
   - $p_1 = p_2$ gives profit of $\frac{D(p_2)}{2}(p_2 - c) > 0$
   - $p_1 = p_2 - \varepsilon$ where ε is positive but small gives $D(p_2 - \varepsilon)(p_2 - \varepsilon - c) > 0$ and for small ε this is strictly higher than above.

   (More technically: $D(p_2 - \varepsilon)(p_2 - \varepsilon - c) > 0 \longrightarrow D(p_2)(p_2 - c)$ for $\varepsilon \longrightarrow 0$)

2. Therefore $p_1 \geq p_2$ can never be a best response for 1, this means we must have $p_1 < p_2$

Step 2b: With $p_1 > c$ it holds in equilibrium that $p_2 < p_1$

- Step 2b follows by the same argument as above, only with switched roles
- Please note that the steps 2a and 2b together imply that there is no equilibrium with $p_1, p_2 > c$
- In equilibrium we must therefore have that $p_1 = c$ and/or $p_2 = c$

Step 3: In equilibrium $p_1 = p_2 = c$

1. Assume that $p_1 = c$ in equilibrium, giving 1 a profit of zero.
2. With $p_2 > c$, $p_1 = p_2 - \varepsilon > c$ would give 1 positive profit, meaning that $p_1 = c$ is not a best response $\Rightarrow$ not an equilibrium
3. With $p_2 = c$, $p_1 > c$ gives a profit of zero (just as $p_1 = c$), but $p_1 < c$ gives negative profit, so $p_1 = c$ is best response
4. Same argument for $p_2 = c$, which shows that the unique equilibrium is $p_1 = p_2 = c$

Intuition

- Step 1: No one will ever set a price below marginal cost, where he/she will lose money with every unit produced.
- Step 2: When my competitor sets a price higher than marginal cost, I can always set a slightly lower price and steal the entire market.
- Step 3: Because of steps 1 and 2, in equilibrium the two firms compete until the equilibrium price is $p_1 = p_2 = c$.

Discussion

- The Bertrand oligopoly model with two firms predicts that both prices will be equal to marginal cost.
- This is the same price as with perfect competition, and from the demand curve it follows that also the quantity is the same, so this equilibrium is *efficient*
- *Much* different to Cournot! In the Bertrand model two firms are enough to have perfect competition and efficiency.

Socrative Quiz Question

True or false: If prices are integer (non-continuous), i.e. if the strategy set is discrete, *both* firms may have positive profits in Nash equilibrium.

Interpretation / Explanation

- Where does this difference in predictions come from? What does this tell us?
- Classical interpretation: When two firms are competing in price this will lead to efficiency, but not when they are competing in quantity.
- There is another intuition behind the difference in predictions. Let's dive deeper into the difference between the two models.

More on differences I

- Imagine a firm which is in one of the two models in equilibrium and what would happen if that firm made a small “error”.

- In the Cournot model, when a firm accidentally produces a bit too much it will lose money since the equilibrium price decreases, but it will still be able to sell what it produces

- In the Bertrand model, when a firm accidentally sets the price a little to high it will lose all of its revenue.

- Mathematically: The result in the Bertrand model is driven by the discontinuous revenue function making a jump.

More on differences II

- One way of thinking about the difference between the two models is therefore to think of the Bertrand model as describing markets in which a price that is too high destroys all revenue.
  - No frictions (such as consumers lacking information about prices / quantities offered, or lack of information on choices of competitor among firms) and completely identical products
  - Fits well for a steel producer selling to firms in a large organized market.
  - Fits less well for a shawarma place which is close to university offering a special dressing.

There are other interesting competition/oligopoly topics

- With oligopoly we may worry about collaboration (agreement on location) with competitors (collusion, cartel, etc.)
  - Possibilities of collaboration between firms can be explored in the context of repeated games with uncertainty.

- In some situations we may have one or few firms in the market but may still have competition due to potential other firms waiting for market entry.
  - We can analyze this in sequential games where existing firms try to scare off potential competitors.

What have we learned?

- The Cournot model with two firms
- The Bertrand model with two firms
- Differences between the two models