UNIVERSITY OF COPENHAGEN

Mikro II, lecture 7a

Price discrimination

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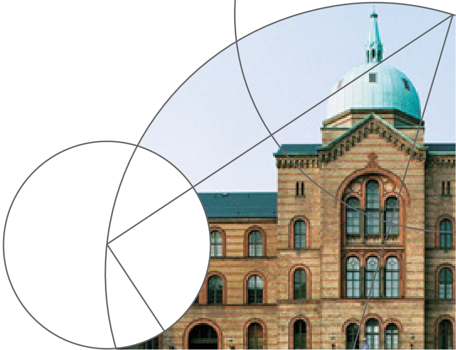


Plan for the lecture

1. First-degree price discrimination
2. Third-degree price discrimination
3. Second-degree price discrimination

Price discrimination

- We have seen that monopoly results in a deadweight loss because the monopolist chooses to produce a lower quantity

- The reason is the negative slope of the demand curve: if the monopolist wants to sell one unit more then that means the price must be lowered on *all* units

- It is crucial here that the monopolist can only set one price for everything it produces

- Our next topic is about what happens if the monopolist can set different prices for different buyers (trades): *price discrimination*

Setup I

- We now consider a monopolist facing some consumers described by (one or more) demand curves

- We will (implicitly) assume that consumers have quasi-linear preferences:
  - Inverse demand curves indicate the willingness to pay for each unit
  - Consumer Surplus measures the willingness to pay if allowed to trade (and can be used directly as a measure of utility)

- We will sometimes interpret the quantity on the market as (partially) reflecting quality instead

Setup II

- We will look at situations where the monopolist may demand different prices from different buyers (details later)
- Important assumption here: Consumers can *not* re-sell what they buy
  - Quite natural for some goods (services in particular): haircuts

First-degree price discrimination

- Let's start by looking at so-called *first-degree* price discrimination
- Assumption: The monopolist has perfect information on all consumers' demand curves / willingness to pay
- The monopolist can set a price for each trade based on the buyer's willingness to pay
- Clearly extreme assumptions about the monopolist's information and trading potential, but useful benchmark

Cost and revenue I

- The monopolist has cost function $C(x)$, marginal cost $MC(x)$, faces (aggregate) demand curve $p(x)$
- What about the revenue function? Think about the marginal revenue $MR(x)$ of selling one unit more if already selling x
  - The marginal willingness to pay for the additional unit follows from the inverse demand curve and must be $p(x)$
  - The monopolist also knows exactly which consumer(s) has this willingness to pay
  - The optimum for the monopolist would therefore be to sell the additional unit at price $p(x)$

Cost and revenue II

- It follows that $MR(x) = p(x)$; what is the overall revenue function?

- Remember that $R'(x) = MR(x)$ and note that by definition, revenue with zero production must be zero; we now get a differential equation with boundary condition:

$$R'(x) = p(x) \quad R(0) = 0$$

- This one has the solution (remember the connection between integrals):

$$R(x) = \int_0^x p(t)dt$$

The monopolist's problem

- The monopolist's problem is

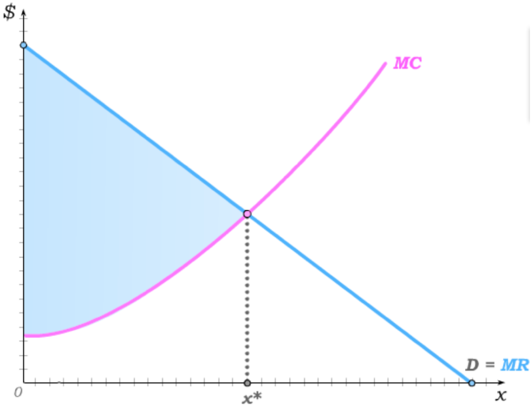
$$\max \quad R(x) - C(x) = \int_0^x p(t)dt - C(x)$$

- FOC is:

$$MR(x^*) = MC(x^*) \iff p(x^*) = MC(x^*)$$

- This is the same as the equilibrium condition in perfect competition (MC = the supply curve here): in the case of first order price discrimination, the traded quantity becomes as in perfect competition $\Rightarrow$ efficient!

First-degree discrimination, graphical



- With first degree price discrimination the amount becomes efficient: no deadweight loss
- The Monopolist’s profit can be found as the area between the MC curve and the demand curve
- Consumer surplus is zero; the monopolist gets all the profits

Socrative Quiz question

What will happen with the price elasticity $\varepsilon(x^*)$ of demand at the optimal quantity when the monopolist introduces price discrimination of the first degree?

- a) It will tend to increase.
- b) It will tend to decrease.
- c) It will stay the same.

Summing up

- In the case of price discrimination of the first degree, the monopolist can take all willingness to pay and run with the entire welfare as profit
- In turn, the inefficiency problem disappears: an extra unit does not lower the price of the other units ...
- ... which means the monopolist chooses to produce until the marginal cost equals the marginal willingness to pay
- The situation thus becomes efficient, but consumers are relatively disadvantaged

Alternative interpretation

- We interpreted the situation as the monopolist selling each item at a specific price
- Alternative interpretation (see Nechyba):
  - The monopolist offers fixed packages to different customers (e.g. 20 units for 100 kr)
  - Or requires a one-time upfront payment to sell at all (e.g., 30 kr plus 10 kr per purchased unit)
  - If the market previously had only one consumer, our solution is to offer her a package of x^* units for a payment of $R(x^*) = \int_0^{x^*} p(t)dt$

Second and third degree, two consumers

- Now we move on to price discrimination of the second and third degree: the monopolist no longer has perfect information about willingness to pay
- Here we will include a further assumption: the monopolist's demand comes from two different types of consumers A and B, each with their (inverse) demand curves, $p_A(x)$, $D_A(p)$ and $p_B(x)$, $D_B(p)$
- The total demand curve is now the horizontal sum of two different demand curves: (remember Micro I / ØP, more on that):

$$D(p) = D_A(p) + D_B(p)$$
$$p(x) = D^{-1}(x)$$

An example without price discrimination

- We will start by talking about third degree price discrimination
- Here we will (among other things) focus on an example where we assume:

$$D_A(p) = \max(12 - p, 0)$$
$$D_B(p) = \max(18 - 3p, 0)$$
$$C(x) = 2 \cdot x$$

- As a benchmark, we will start by solving the monopolist's problem here without price discrimination

The aggregate demand functions

- We start by finding the total market demand; (horizontal) addition of the two demand functions:

$$D(p) = \begin{cases} 12 - p & \text{for} \quad 6 \le p \le 12 \\ 30 - 4 \cdot p & \text{for} \quad 0 \le p < 6 \end{cases}$$

- From here we can find the total inverse demand curve which becomes:

$$p(x) = \begin{cases} 12 - x & \text{for} \quad 0 \le x < 6 \\ 7.5 - \frac{1}{4} \cdot x & \text{for} \quad 6 \le x \le 30 \end{cases}$$

- The demand functions here are continuous and linear, but slightly ugly: not differentiable at $x = 6$ $(p = 6)$

The monopolist's problem

- Our monopolist maximizes profit:

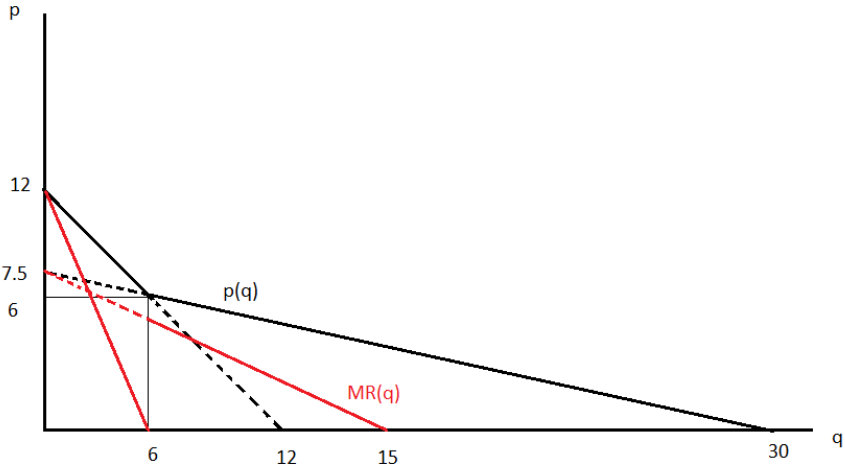
$$\max \quad R(x) - C(x) = p(x)x - 2 \cdot x$$

- Pitfall: the profit function here is not concave and the first order condition will yield more than one solution

- The kinked demand function leads to a "jump" in MR at $x = 6$ from 0 to 4.5 (remember that marginal revenue is $p'(x)x + p(x)$):

$$MR(x) = \begin{array}{ll} 12 - 2 \cdot x & \text{for} \quad 0 \leq x < 6 \\ 7.5 - \frac{1}{2} \cdot x & \text{for} \quad 6 \leq x \leq 30 \end{array}$$

Inverse demand and marginal revenue, graphically



Solution

- First order condition $MR(x) = 2$ is met at two places: $x = 5$ and $x = 11$
- Checking the second order condition reveals that both of these are local maxima
- We find the global maximum by calculating and comparing profits:
  - $x = 5$ yields the price 7, costs of 10 and profit of 25
  - $x = 11$ yields the price 4.75, costs of 22 and a higher profit of 30.25, hence this is the optimum

Socrative Quiz Question

What happens with the revenue of the monopolist if the WTP among consumers of group A for the first unit drops from 12 to 10? Think about which group(s) of consumers are served along which part of the demand curve.

- a) It increases
- b) It decreases
- c) It stays the same

Third-order price discrimination

- Now we give the monopolist the opportunity to do price discrimination of the third order

- The monopolist can see the difference between type A and type B consumers and can set different prices for the two groups

- The monopolist also knows how the demand curves of the types look overall, but different to the case of price discrimination of the first degree, the monopolist cannot determine the willingness to pay for each unit and buyer

- Therefore, the monopolist cannot set a different price for each unit sold or offer each consumer a ”fixed package” with a specific quantity

The monopolist's problem

- The monopolist must now choose the price to be offered to each type of consumer, formally stated:

$$\max_{p_A, p_B} p_A \cdot D_A(p) + p_B \cdot D_B(p) - C(D_A(p) + D_B(p))$$

- 1-1 relationship between price and quantity for each type: The problem is equivalent to choosing the quantities instead:

$$\max_{x_A, x_B} p_A(x_A) \cdot x_A + p_B(x_B) \cdot x_B - C(x_A + x_B)$$

- FOCs are:

$$p_A(x_A^*) + p_A'(x_A^*)x_A^* = MC(x_A^* + x_B^*)$$
$$p_B(x_B^*) + p_B'(x_B^*)x_B^* = MC(x_A^* + x_B^*)$$

The FOCs

$$p_A(x_A^*) + p_A'(x_A^*)x_A^* = MC(x_A^* + x_B^*)$$
$$p_B(x_B^*) + p_B'(x_B^*)x_B^* = MC(x_A^* + x_B^*)$$

- First order conditions are similar to standard monopoly, only now there are two: MR of type A must be equal to MC and the same goes for type B
- It follows that the markup formula from slides 6b also holds here: the price for each group is a markup over the marginal cost, where the markup depends on demand elasticity (of that group)

An easy special case

- The situation becomes particularly easy if we assume constant marginal costs $MC(x_A + x_B) = c$
- In that case, only the quantity for type A consumers is included in the corresponding first-order condition and the same applies for type B consumers (no longer both quantities in both conditions):

$$p_A(x_A^*) + p_A'(x_A^*)x_A^* = c$$
$$p_B(x_B^*) + p_B'(x_B^*)x_B^* = c$$

- In that case, it is just like solving two completely separate standard monopoly problems

Example from before I

- Let's look at the example from before, where we assumed constant marginal costs:

$$D_A(p) = \max(12 - p, 0)$$
$$D_B(p) = \max(18 - 3p, 0)$$
$$C(x) = 2 \cdot x$$

- The inverse demand curves are:

$$p_A(x) = 12 - x \quad \text{for} \quad 0 \leq x \leq 12$$
$$p_B(x) = 6 - \frac{1}{3} \cdot x \quad \text{for} \quad 0 \leq x \leq 18$$

Example from before II

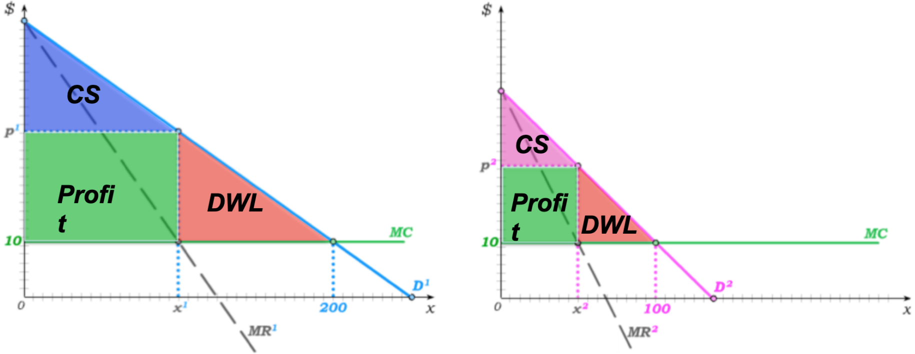
- The first-order conditions for each type set the MR equal to the marginal cost:

$$12 - 2 \cdot x_A^* = 2 \iff x_A^* = 5$$
$$6 - \frac{2}{3} \cdot x_B^* = 2 \iff x_B^* = 6$$

- We thus get $p_A = 7$, $p_B = 4$ and a total profit of 37 (without price discrimination we got the price 4.75 and the profit 30.25)

- Third-order price discrimination allows a lower price for the price sensitive B consumers and a higher price for the less price sensitive A consumers $\Rightarrow$ higher profits

Third-degree price discrimination still causes DWL



(NB Nechyba's example uses different numbers!)

- Third degree price discrimination still causes a deadweight loss and it may be greater or less than without price discrimination

Third order price discrimination with rising MC

- If we do not have constant marginal costs, then third-order price discrimination becomes a little more complicated because the quantity sold for type A affects MC of producing also for B

- We now have two equations with two unknowns:

$$p_A(x_A^*) + p_A'(x_A^*)x_A^* = MC(x_A^* + x_B^*)$$
$$p_B(x_B^*) + p_B'(x_B^*)x_B^* = MC(x_A^* + x_B^*)$$

- to exemplify, we can recalculate the previous example with changed cost function $C(x) = \frac{1}{2}x^2$ so $MC(x) = x$

Example with increasing MC

- Insert for MR in first order conditions and for MC and isolate respectively x_A^* and x_B^*

$$12 - 2 \cdot x_A^* = x_A^* + x_B^* \iff 12 - 3 \cdot x_A^* = x_B^*$$
$$6 - \frac{2}{3} \cdot x_B^* = x_A^* + x_B^* \iff 6 - \frac{5}{3} \cdot x_B^* = x_A^*$$

- Insert for x_B^* in the second equation and then find x_A^*

$$6 - \frac{5}{3} \cdot \left(12 - 3 \cdot x_A^*\right) = x_A^* \iff x_A^* = 3.5$$

- Inserting into the first equation finally gives $x_B^* = 1.5$ and from the demand curves follows $p_A^* = 8.5$ and $p_B^* = 5.5$

Socrative Quiz Question

True or false: In the previous example, if the inverse demand curve becomes flatter / rotates outward (i.e. overall more price elastic) for one group of consumers, then, holding everything else constant, the quantity offered to the other group in equilibrium will tend to decrease.

Summing up

- For third-degree price discrimination, the monopolist increases her profits by offering different groups of consumers different prices according to their willingness to pay (demand elasticity)

- This is only possible because we have assumed that the monopolist can see the difference between the consumer groups (and has the opportunity to offer different prices)

- Third degree price discrimination is widely used in practice: youth / pensioner tickets, student discounts, etc.

Second degree price discrimination I

- We finally consider second degree price discrimination
- Here we assume that the monopolist *cannot* see a difference between the A and B consumer groups
- On the other hand, the monopolist can offer her goods in fixed packages where you can only buy a certain quantity at a certain price
- Here we assume that each of the consumer groups consists of a number of consumers with identical demand; for the sake of convenience, we assume that there is only one consumer of each type

Second degree price discrimination II

- Thus, in the case of second-degree price discrimination, the monopolist can try to increase her profits by offering various packages that try to exploit the willingness to pay for the two consumers ...

- ... but the monopolist can't see the difference between the two consumers, so can't control which package they choose to buy

- But that sounds like something we've seen before ...

- Yes! One can analyze the situation here as a Principal Agent problem where the principal (monopolist) does not have information on the types of the two agents (consumers)

Principal Agent

- Our monopolist must therefore offer two packages intended for the A consumer and B consumer respectively; each consists of a quantity of x_A or x_B and a corresponding total price of S_A or S_B

- Here the monopolist's profits will be (assuming the packages are purchased by the intended consumers):

$$S_A + S_B - C(x_A + x_B)$$

- Profit should (as usual) be maximized given some constraints of consumer choice; In the past we did it via utility functions, but here we have only assumed demand curves ...

From demand to utility comparison

- We can formulate the constraints by looking at consumer surplus (remember we have assumed quasi-linear preferences)

- Let $CS_A(x)$ indicate the consumer surplus for consumer A if he receives the amount of x without paying anything (at a cost per unit of 0):

$$CS_A(x) = \int_0^x p_A(t)\,dt$$

- Now it will apply:
  - $CS_A(x)$ indicates the maximum consumer A will pay upfront for a deal where he receives x units without having to pay anything per piece in addition
  - If we give consumer A the choice to either pay a lump sum S and then receive x units, or to pay a lump sum $\bar{S}$ and then receive $\bar{x}$ then he will prefer the first option if $CS_A(x) - S \geq CS_A(\bar{x}) - \bar{S}$

IC and IR conditions

- We accordingly define $CS_B(x) = \int_0^x p_B(t)\,dt$ for consumer B, and can now write down the terms of our Principal Agent problem

- IR conditions guarantee that the two consumers will choose to purchase their respective packages rather than not buying anything:

$$CS_A(x_A) \geq S_A \qquad (\text{IR}_A)$$

$$CS_B(x_B) \geq S_B \qquad (\text{IR}_B)$$

- The IC conditions ensure that consumers are willing to choose the package that is intended for them:

$$CS_A(x_A) - S_A \geq CS_A(x_B) - S_B \qquad (\text{IC}_A)$$

$$CS_B(x_B) - S_B \geq CS_B(x_A) - S_A \qquad (\text{IC}_B)$$

Principal Agent problem

- A monopolist using second-degree price discrimination thus solves the following problem:

$$\max_{S_A, x_A, S_B, x_B} \quad S_A + S_B - C(x_A + x_B)$$

$$\text{s.t.}$$

$$CS_A(x_A) \geq S_A \qquad (\text{IR}_A)$$

$$CS_B(x_B) \geq S_B \qquad (\text{IR}_B)$$

$$CS_A(x_A) - S_A \geq CS_A(x_B) - S_B \qquad (\text{IC}_A)$$

$$CS_B(x_B) - S_B \geq CS_B(x_A) - S_A \qquad (\text{IC}_B)$$

A simple example I

- Except for new notation (and a different profit function), this problem is very similar to our previous Principal Agent problems

- We therefore won't go into more detail with the math (See problem set 8 for a mathematical example or Nechyba for a thorough graphical analysis that does not explicitly use the Principal Agent framework)

- Instead, we will briefly describe and discuss the solution in a specific case where consumer A has a high willingness to pay relative to B and the marginal cost of the monopolist is 0:

$$p_A(x) = 12 - x_A$$
$$p_B(x) = 8 - x_B$$
$$C(x) = 0$$

A simple example II

- Benchmark: first solve without the IC conditions, the monopolist takes the consumers' full willingness to pay and they get zero consumer surplus:

$$x_A = 12, \quad S_A = 72$$
$$x_B = 8, \quad S_B = 32$$

- This is very similar to first degree price discrimination and efficient, but these two packages do not meet the IC conditions: Consumer A can get positive CS by buying B's package
- If you solve the full problem you will instead find that the optimal solution is the following packages:

$$x_A = 12, \quad S_A = 56$$
$$x_B = 4, \quad S_B = 24$$

A simple example III

$$x_A = 12, \quad S_A = 56$$
$$x_B = 4, \quad S_B = 24$$

- Note the clear pendant to our example of environmental regulation (and with the same intuition):
  - It would be optimal (and efficient) to sell the packages $x_A = 12$, $S_A = 72$, $x_B = 8$, $S_B = 32$, but then A will choose B 's package
  - Therefore, it ends up being optimal to make B's package a bit inferior (lower quantity) and cheaper ...
  - ... and at the same time making A's package slightly cheaper $\Rightarrow$ ensures that A chooses ”right”

Socrative Quiz Question

True or false: If the fraction of B consumers becomes high, it may become optimal to only offer one package of a quantity of 4 and at a price of 24.

Summing up

- In the case of second-degree price discrimination, the monopolist increases her profits by bundling his products into different packages

- However, to separate the different groups (ensure that the groups “choose correctly”), a deadweight loss will occur (one of the groups will receive an inefficiently bad package)

- Second-degree price discrimination is also seen in practice: e.g. business class vs. economy on flights (x is the quality of the flight), VIP tickets for events

- Also note that the model points to incentives to make economy class extra bad (!)

What have we learned?

- The effects of different kinds of price discrimination
- To calculate the outcome of first degree price discrimination
- Solve problems with third degree price discrimination
- Put second degree price discrimination into a Principal Agent framework