Mikro II, lecture 5a

Adverse selection in the labor market

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Plan for the lecture

1. Model of adverse selection in the labor market
2. Signaling/screening in the labor market
3. Nechyba 22B revisited

Adverse selection I

- We now dive into the second Sloth note which focuses on *Adverse Selection*:
  - Asymmetric information: One side of the market knows something about themselves that the other side doesn't know
  - The uninformed side of the market can thus have trouble attracting (selecting) the right agents ...
  - ... and the prices offered (contract terms) may end up attracting "the wrong ones" (adverse)

Adverse selection II

- We have already seen a few examples in lecture 4a: used cars (Akerlof) and non-existent insurance markets
- We will now look at adverse selection in the labor market ...
- ... and in the context of a Principal-Agent problem
- Principal: Employer; Agent(s): Worker(s)

Workers and output

- There are two types of workers in the labor market: a share of q have high productivity H, a share of $(1-q)$ have low productivity L; if the workers are employed by the principal they produce a quantity of output

- As something new (and slightly different to Sloth), workers can choose to do extra work $e > 0$ (effort) which can increase the quality and value of the output; the output of the L and H workers, respectively, is given by:

$$py_L + \alpha \cdot e \qquad \text{and} \qquad py_H + \alpha \cdot e$$

- If the workers do not bother ($e = 0$) the value of the output is py_L and py_H, otherwise the value of output increases by α for each additional unit of ”effort”.

Utility

- Workers' utility depends positively on the wages they receive and negatively on how much effort they provide:

$$u_H(w,e) = w - b_h \cdot f(e)$$

$$u_L(w,e) = w - b_l \cdot f(e)$$

$$f(0) = 0, f' > 0, f'' > 0,\ b_l > b_h$$

- The function f: the more effort the workers provide, the greater the cost of utility and the marginal cost will be growing

- The constants $b_l > b_h$: Low-productivity workers have a higher cost of doing good than high-productivity workers (both overall and on the margin)

Hiring process (Principal-agent)

- The employer meets with a random worker
- The employer acts as principal and offers a contract to the worker with a wage offer w
- The worker can say no to the contract; outside option for high productivity worker r_H, outside option for low productivity worker r_L
- We assume that high types have better outside options than low types: $r_H > r_L$

A specific scenario

- Like Sloth, we start by analyzing a situation where the agent is never asked (and never chooses) to provide extra effort, i.e. we assume that $e = 0$ (i.e. the output is py_L or py_H and the worker's utility is equal to the salary w)

- In addition, Sloth splits her analysis into four cases, depending on whether the following two conditions hold
  - $py_H > r_H$, so high-productivity people say yes to a wage that is lower than their productivity (socially efficient that they are hired)
  - $py_L > r_L$, so low-productivity people say yes to a wage that is lower than their productivity (socially efficient that they are hired)

- We start by assuming that both conditions are met

Information

- We need to clarify the information structure of the model: who knows what?
- We start by assuming that the workers know whether they are high or low productivity ...
- ... and that employers can also immediately determine a worker's productivity

Who will the principal try to hire?

- An important choice for the principal in this model is who she will try to hire: Only H? Only L? Both? None?
- As before, we will do our analysis for each scenario, solve for the optimal contract in that case, and compare the options afterwards
- Given our assumptions earlier, it seems intuitive that the principal will try to hire both types so we start with that scenario.

The principal's problem, full information

- The principal observes the type of workers $\Rightarrow$ can condition the salary on the type: two contracts with different wages w_H, w_L

- The principal maximizes expected profit (probability q of worker being high type):

$$\max_{w_H, w_L} \quad q\left(py_H - w_H\right) + (1-q)\left(py_L - w_L\right)$$

$$\text{s.t.}$$

$$w_H \geq r_H \qquad (\text{IR}_H)$$

$$w_L \geq r_L \qquad (\text{IR}_L)$$

Solution, full information

- The IR conditions bind obviously (otherwise lower the salary) so we get:

$$w_H = r_H \qquad \text{and} \qquad w_L = r_L$$

- Both types are offered exactly their outside option (reservation utility)

- Assuming $py_H > r_H$ and $py_L > r_L$, the principal makes a positive profit on both so easy to check that the principal WILL employ both

- Fairly obvious outcome given our assumptions (profitable to hire both types, full information)

Amendment 1: Asymmetric information

- Now we assume that the firm cannot observe the type of a worker and therefore cannot condition the contract on the type

  (not clear from the job seeker's CV and difficult to determine individual contributions in total production)

- Again, the principal must choose who she wants to try to hire; we look again at the situation where she tries to hire both
- As before, different terms (different contracts) are offered for the two types, but now we have to make sure that the different types themselves choose "the right" contract
- This introduces IC side conditions for the two types

The principal's problem, asymmetric information

$$\max_{w_H, w_L} \quad q\left(py_H - w_H\right) + \left(1-q\right)\left(py_L - w_L\right) \quad \text{s.t.}$$

$$w_H \geq r_H \qquad (\text{IR}_H)$$

$$w_L \geq r_L \qquad (\text{IR}_L)$$

$$w_H \geq w_L \qquad (\text{IC}_H)$$

$$w_L \geq w_H \qquad (\text{IC}_L)$$

- Silly problem! The IC conditions clearly mean that $w_H = w_L$, so in practice only one salary can be offered
- The principal cannot see the difference so he cannot offer different contracts to the different types

The principal's problem, one wage level

$$\max_{w} \quad q\,(py_H - w) + (1-q)\,(py_L - w)$$

$$\text{s.t.}$$

$$w \geq r_H \qquad (\text{IR}_H)$$

$$w \geq r_L \qquad (\text{IR}_L)$$

- We have assumed $r_H > r_L$ so that IR$_H$ binds, solution: $w = r_H$
- If the highly productive should say yes then the salary must match their (high) reservation salary

Other hiring strategies I

- Is this the optimal contract or will the principal perhaps choose something other than hiring both?
  - If $w = r_H > py_L$ the principal gets a negative profit every time they hire (meet) a low-productivity worker
  - However, if low productivity is rare (q large), overall profit may still be positive and it may be optimal to hire both at the same wage.

- Alternative contracts
  - Do not hire any of the types (set the salary $w < r_L < r_H$): yields zero profit
  - Hire only the high types: Impossible! if the salary is high enough for H to say yes then L also says yes (the constraints of the formal problem have no solution)
  - Hire only the low-productive: Optimal salary is $w = r_L$, with guaranteed positive profit because $py_L > r_L$

Other hiring strategies II

- The optimal choice is either to hire everyone at $w = r_H$ or to hire only the low-productive at $w = r_L$; depends on what gives higher profit:

$$q\,(py_H - r_H) + (1-q)\,(py_L - r_H) \qquad \text{vs.} \qquad (1-q)\,(py_L - r_L)$$

- Trade-off: Hire only the low types at low wage or hire everyone at a higher wage

- Depending on the parameters, one or the other may be optimal

Socrative Quiz Question

True or false: If the price of the produced good p increases, it will become more likely that the principal chooses to hire both.

Other cases

- If $py_H < r_H$ and $py_L < r_L$ the solution is to not hire anyone (offer salary of 0)
- If $py_H < r_H$ and $py_L > r_L$ the solution is to only employ the low types at their reservation wages
- If $py_H > r_H$ and $py_L < r_L$, the solution will depend on the parameters:
  - May be optimal to hire everyone at the high salary (and accept negative profits on low types)
  - May be optimal not to hire anyone (if high wages attract too many low types)

Adverse selection

- The model illustrates that adverse selection can cause parts of the labor market to collapse; the high-productivity market may shut down because a wage offer at high types' high reservation wages can attract too many less productive
- Direct pendant to Akerlof's used cars (lemons) from earlier
- As before: *information costs* for the principal, but also an *information rent* for low-productivity agents (if everyone is hired, it will be at the high salary)

Signaling and screening

- We previously talked about asymmetric information providing incentives to signal and / or screen
- In Nechyba's model for asymmetric information, we talked about very explicit screening / signaling; now more implicitly
- To study this, we will now allow employees to make an extra signaling effort ($e > 0$)

Contracts with efforts

- A wage contract will now involve two things: a wage w and a level of effort e

- Important detail: How do we assume that e can be included in an employment contract?

- You can argue for different things; we will assume that e is observed and can be conditioned on in the contract

- So we have to think about the extra effort as something the employer can objectively decide

  (e.g. the complexity of the tasks assigned, but much more on that later!)

Reminder

- Output now depends on both the type and the effort level of the worker

$$py_L + \alpha \cdot e \qquad \text{og} \qquad py_H + \alpha \cdot e$$

- The utility of the workers depend positively on the salary they receive and negatively on how much effort they have to make:

$$u_H(w,e) = w - b_H \cdot f(e)$$

$$u_L(w,e) = w - b_L \cdot f(e)$$

$$f(0) = 0, f' > 0, f'' > 0, b_L > b_H$$

The principal's problem

- We maintain that there is asymmetric information about the types, but assume that both types are employed; The principal's problem is to choose two payment levels w_H, w_L and two effort levels e_H, e_L:

$$\max_{w_H, e_H, w_L, e_L} \quad q\left(py_H + \alpha e_H - w_H\right) + (1-q)\left(py_L + \alpha e_L - w_L\right)$$

s.t.

$$u_H(w_H, e_H) \geq r_H \qquad (\text{IR}_H)$$

$$u_L(w_L, e_L) \geq r_L \qquad (\text{IR}_L)$$

$$u_H(w_H, e_H) \geq u_H(w_L, e_L) \qquad (\text{IC}_H)$$

$$u_L(w_L, e_L) \geq u_L(w_H, e_H) \qquad (\text{IC}_L)$$

Graphical analysis

- This Principal Agent problem can be most conveniently analyzed graphically in an (e, w) diagram

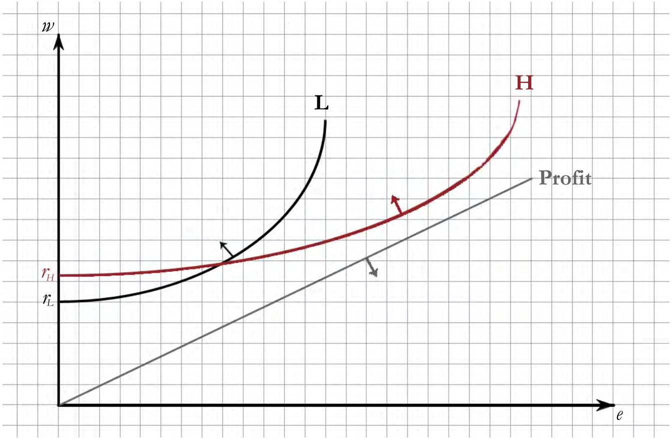
- The agent's indifference curves corresponding to their reservation utility are:

$$u_H(w,e) = r_H \iff w - b_H f(e) = r_H \iff w = r_H + b_H f(e)$$
$$u_L(w,e) = r_L \iff w - b_L f(e) = r_L \iff w = r_L + b_L f(e)$$

- Isoprofit curves for the principal for each type of worker are:

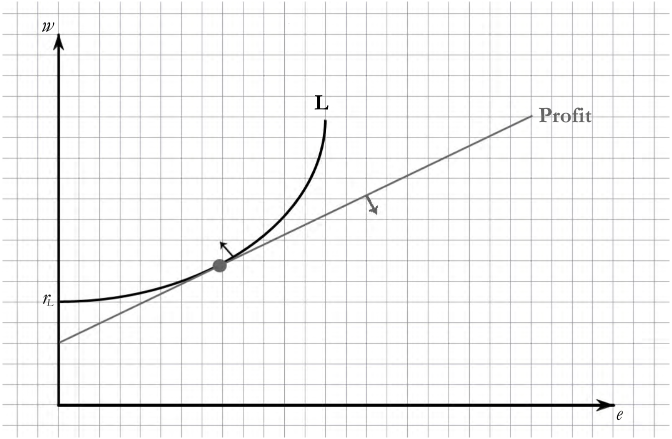
$$py_H + \alpha e - w = \bar{\pi}_H \iff w = py_H - \bar{\pi}_H + \alpha e$$
$$py_L + \alpha e - w = \bar{\pi}_L \iff w = py_L - \bar{\pi}_L + \alpha e$$

Curves and utility / profit directions



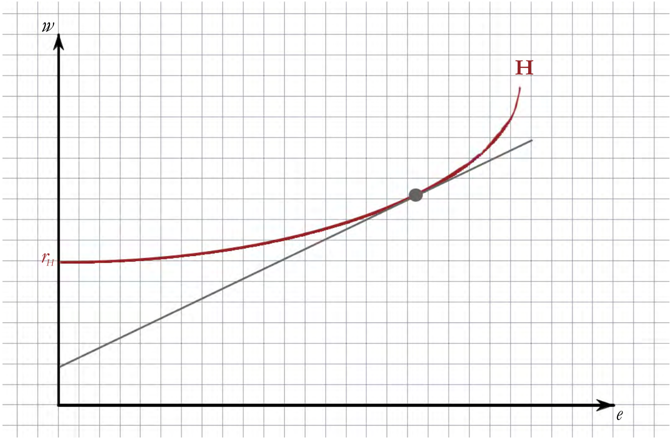
- Indifference curves are convex functions and the utility grows upwards to the left; isoprofit curves are linear and profits grow downwards to the right.

Assume only low-productivity workers ($q = 0$)



- Optimal contract (w, e) is the tangency between the low-productivity isoprofit curve and the indifference curve corresponding to the outside option

Assume only high productivity workers ($q = 1$)



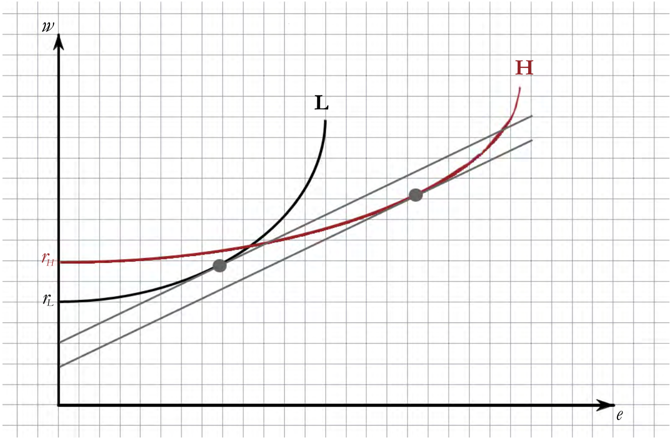
- Optimal contract (w, e) is the tangency between high-productivity isoprofit curve and the indifference curve corresponding to the outside option

Socrative Quiz Question

Assume there are only high productivity workers, and we start with a contract in which the firm produces and makes positive profit. What would happen if the function $f(e)$ now changes from having $f''(e) > 0$ to having $f''(e) = 0$?

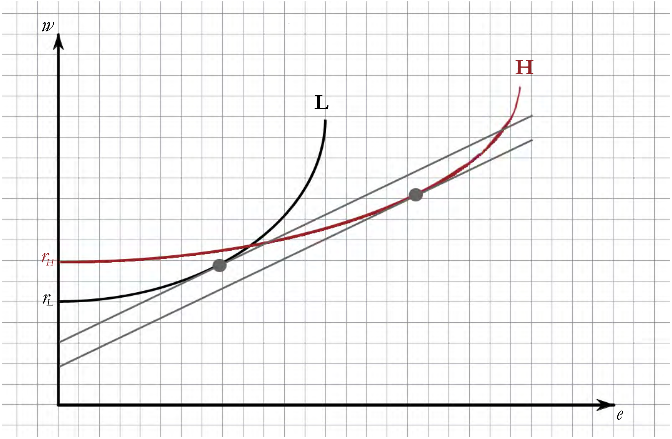
- a) It would become optimal to require zero effort.
- b) It would become optimal to require infinite effort.
- c) It would become optimal to require zero or infinite effort depending on the other parameters of the problem.
- d) The market would break down and there would be zero production.
- e) It would become optimal to require zero or infinite effort, or have no production, depending on the other parameters of the problem.

Optimal contract I



- Is it a solution to the overall problem of offering the two contracts from the previous slides?

Optimal contract I

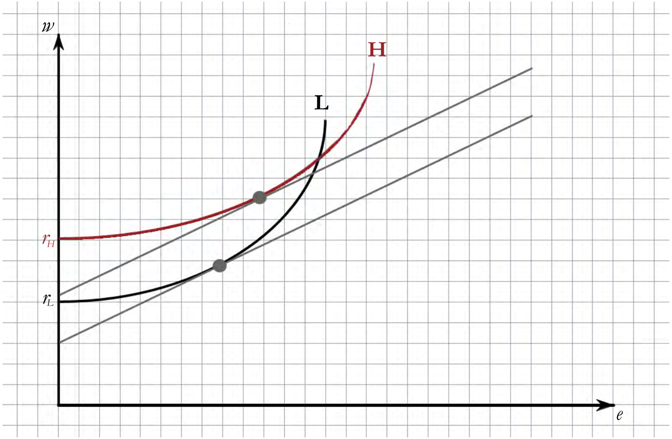


- Is it a solution to the overall problem of offering the two contracts from the previous slides? Yes!

Optimal contract II

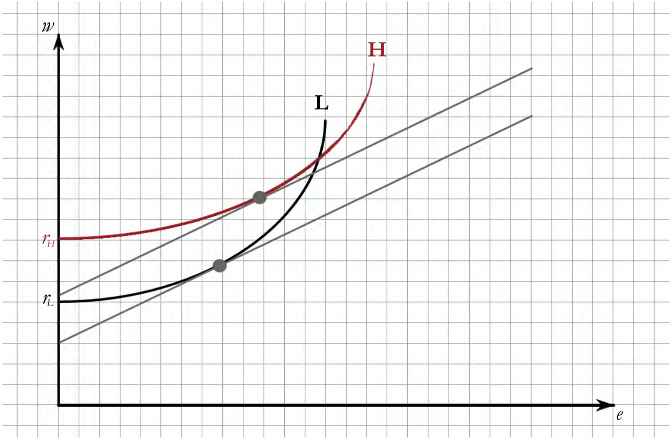
- The offered contracts are the most profitable that comply with the IR conditions (located on / above the indifference curves)
- IC conditions are also met: the contract for H is below L indifference curve and vice versa (so IC does not bind)
- When the contracts can condition on effort levels, the problem of adverse selection can be overcome
- The reason is that effort is more expensive for the low-productive than the high-productive so effort acts as screening / signaling

Another situation I



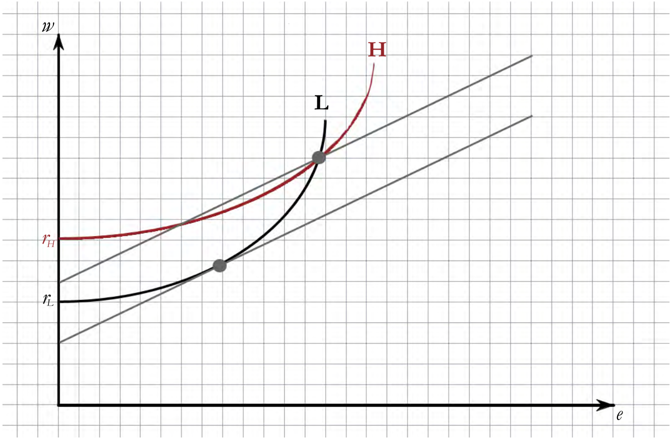
- Are these two contracts a solution to the Principal's problem?

Another situation I



- Are these two contracts a solution to the Principal's problem? No, L strictly prefers to choose H's contract

Another situation II



- If IC_L is to be complied with, H's contract may require more effort but provide higher payment (alternatively, L's contract must be improved)

Another situation II

- In this new situation, IC_L binds: H's contract lies at the intersection of two indifference curves

- Again, there is an information cost for the principal (lower profit than under full information); the outcome is *not* efficient: H's contract requires "too much" effort

- In order to achieve the optimal contract mathematically, we also have to:
  - Take a look at the Principal's profit to see if it is optimal to change H's contract, change L's contract, or a combination

  - Check that the company does not want to offer only a single contract that is optimal for one of these types (compare profits)

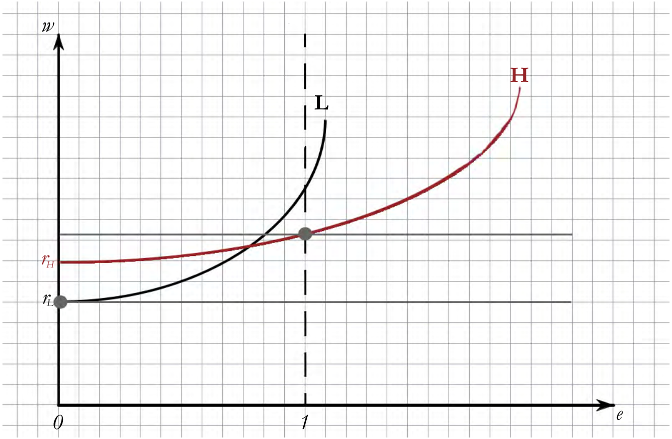
Socrative Quiz Question

True or False: In the previous example no one earns information rents.

Sloth's version I

- Sloth's model of effort is a special case of the one we just saw
- Sloth's model is equivalent to assuming that e can only take two values e.g. $e = 0$ or $e = 1$
- Sloth also assumes that efforts do not increase revenue: $\alpha = 0$
- (In addition, Sloth's model is equivalent to assuming $f(1) = 1$ in the slide notation)

Sloth's version II



- The isoprofit curves become flat ($\alpha = 0$) and the contracts can only be on the y-axis or the dotted vertical line

Sloth's version II

- If the indifference curves are correct, as before, we can find a separating contract where H provides extra effort

- We can elaborate ”are correct” and translate into math (see Sloth for more):
  - Graphically, a separating contract will be found if L's indifference curves intersect the vertical line over H's indifference curve

  - The intersections of the two indifference curves with the vertical line at $e = 1$ are $r_L + b_L$ and $r_H + b_H$ so this is the case if $r_L + b_L > r_H + b_H$

- The inefficiency becomes very clear: extra effort has *no effect* on production, but is required by the high types solely as a signal

e stands for...

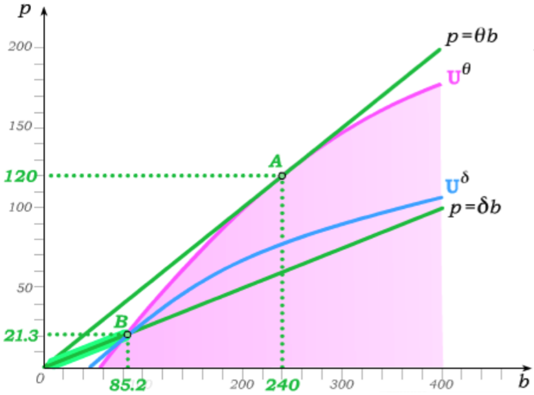
- We have interpreted e as effort but e can be reinterpreted as education:
  - The employer offers contracts that (may) require a certain level of education
  - High and low-productivity workers have different costs in completing an education
  - In the separating contract, high-productivity people choose to complete a demanding education and therefore receive a high-wage contract

Education as signaling

- The model with e as education comes from Michael Spence (Nobel Prize 2001)
- Education as signaling:
- Education helps people who are already of high productivity in signaling to employers that they are
- Note: Works if education also makes you more productive ($\alpha > 0$), but also if education doesn't matter ($\alpha = 0$)

Nechyba 22B revisited

- We skipped Nechyba section 22B in the lecture
- Graphic: Company makes insurance for 2 risk types
- Close up of these slides:
  - Utility / profit directions are the other way around
  - Actuarial fair price instead of take-it-or-leave-it offer (zero profit rather than positive profit)



What have we learned?

- How adverse selection can look in the labor market
- How signaling/screening can solve the problems of adverse selection
- Example of graphical analysis of the Principal Agent problem with two types of agents
- Training as signaling
- More about what adverse selection can look like in the insurance market (Nechyba 22B)