Mikro II, lecture 3b

Asymmetric information: Moral hazard in insurance contracts

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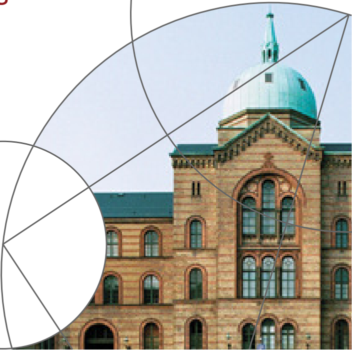

Plan for the lecture

1. Principal Agent model for constant risk insurance
2. Principal Agent model for insurance if the agent can influence the risk
3. Principal Agent model for insurance with moral hazard
4. Link between Principal Agent models for insurance and for the labor market

Moral Hazard

- We now focus more closely on moral hazard issues associated with various contracts
- Moral hazard: one party has the opportunity to do something that the other does not want after the two have signed a contract
- Moral hazard occurs when one party's action cannot be observed and / or cannot reasonably be specified in a contract (can be seen as a type of asymmetric information)
  - It is non-observable whether you daydream during working hours
  - It is not realistic that we can tie you to a contract that you should *do your best*

Principal Agent model for insurance I

- The agent is a consumer whose utility depends on consumption (c) via the utility function $v(c)$; consumption depends on income M and whether the agent experiences an ”accident”:
  - With probability π the accident will occur and the agents loses L, that is: $c = M - L$
  - With probability $1-\pi$ there is no accident, that is: $c = M$

- The principal is an insurance company that sells insurance with price Γ and payout K; with insurance, the agent pays Γ and then:
  - With probability π, the agent experiences an accident and loses L but gets K, i. e. $c = M - \Gamma - L + K$
  - With probability $1-\pi$, the agent does not experience an accident, ie. $c = M - \Gamma$

Principal Agent model for insurance II

- The principal offers the agent an insurance contract as a take-it-or-leave it offer; an insurance contract is determined by a price (insurance premium), Γ, and its payment (insurance sum), K

- The agent may choose to say no and thus proceed without insurance; reservation utility / outside option:

$$\bar{v} = \pi \cdot v(M - L) + (1 - \pi) \cdot v(M)$$

- We have assumed that the probability of an accident is fixed (more on that later) and assume that this is known by everyone

Risk aversion

- We assume (as always) that the agent maximizes expected utility. We also assume that $v' > 0$ and $v'' < 0$, i.e. the consumer likes (as always) consumption and is *risk averse* (realistic)
- For the insurance company (and companies in general) we assume (as always) that it maximizes expected profits:
  - This means that the insurance company behaves risk-neutral (only expected profit matters)
  - Often motivated by the insurance company dealing with many agents
  - Law of large numbers: the proportion of customers who have an accident goes against a fixed number as the number of customers grows (risk disappears)
- A risk averse agent vs a risk neutral company: intuitively there is great potential for trading risk (insurance)

Principal Agent problem I

- The principal maximizes expected profits: Profit if accident occurs is $\Gamma - K$, profit with no accident is Γ

$$\max_{\Gamma,K} \quad \pi \cdot (\Gamma - K) + (1-\pi) \cdot \Gamma$$

$$\text{s.t.}$$

$$\pi \cdot v(M - \Gamma - L + K) + (1-\pi) \cdot v(M - \Gamma) \geq \bar{v} \qquad (\mathrm{IR}_0)$$

- Note: It is assumed that only two things can happen (accident / no accident), so the contract only consists of two numbers Γ, K which we choose such as to maximize expected profits

Rewriting

- Let $c_1 = M - \Gamma$ be consumption without accident and $c_2 = M - \Gamma - L + K$ be consumption with accident
- Now note that if we choose a particular Γ and K then we can derive c_1 and c_2 directly
- Conversely, if we choose that we want a certain c_1 and c_2 then we can derive what Γ and K must be
- We can therefore rewrite the problem and "pretend" that the contract specifies c_1 and c_2 instead of Γ and K

Principal Agent problem II

- Rewrite and combine the definition of c_1, c_2 to get an expression for Γ and K:

$$\Gamma = M - c_1$$
$$K = c_2 - M + \Gamma + L = c_2 - c_1 + L$$

- Use these to rewrite the principal's problem:

$$\max_{c_1, c_2} \quad M - \pi \cdot L - (1-\pi)c_1 - \pi c_2$$

$$\text{s.t.}$$

$$\pi \cdot v(c_2) + (1-\pi) \cdot v(c_1) \geq \bar{v} \qquad (\text{IR}_0)$$

Solution to the problem

- Easy to realize that the constraint must bind (otherwise increase the price) so we can solve by Lagrange:

$$L(c_1, c_2, \lambda) = M - \pi \cdot L - (1-\pi)c_1 - \pi c_2 + \lambda\left(\pi \cdot v(c_2) + (1-\pi) \cdot v(c_1) - \bar{v}\right)$$

- FOCs:

$$\begin{aligned} -(1-\pi) + \lambda(1-\pi)v'(c_1) = 0 &\iff v'(c_1) = \tfrac{1}{\lambda} \\ -\pi + \lambda \pi v'(c_2) = 0 &\iff v'(c_2) = \tfrac{1}{\lambda} \end{aligned}$$

- $v'' < 0 \Rightarrow v'$ strictly decreasing so that it follows that $c_1 = c_2$ for the optimal contract (ie $K = L$, from the constraint we could find Γ as well)

Full insurance

- We see that the optimal contract entails full insurance (remember that this is also pareto-optimal), i. e. there is *no deductible*

- Intuition: The agent doesn't like risk, the company doesn't care about risk $\Rightarrow$ efficient that the company takes over all the risk

- Question: The maximization problem maximized the company's profit, so why do we end up with a contract that helps the agent with his risk?

- Answer: Providing the agent with the type of insurance he wants will increase his willingness to pay for the contract and thus the profit of the insurance company

Socrative Quiz Question

What happens with the profits of the company if the agent becomes more risk-averse?

- a) They increase
- b) They decrease
- c) They stay the same
- d) They increase or decrease depending on the size of the loss L

Amendment 1: The agent controls the risk

- We now change the model: After the contract is signed, the agent must make a choice (first step towards moral hazard):
  - The agent can be careless, and the risk of accident is π_s
  - The agent can be cautious, which imposes costs of $e > 0$ to the agent, but lowers the risk of accident to $\pi_f < \pi_s$

- Note: The outside option is determined by the agent's utility without insurance (see Sloth note for more analysis of this choice):

$$\max\left(\pi_f v(M-L) + (1-\pi_f)v(M) - e \quad , \quad \pi_s v(M-L) + (1-\pi_s)v(M) \right) = \bar{v}$$

- We still assume that everyone knows the risk and the behavior of the agent.

Contract

- We will also start by assuming that the insurance company can observe if the agent is cautious.

  (Think of a bike insurance that only covers if you have an approved lock)

- Two possible things the insurance company can do:

  1. Offer a contract that requires the agent to be careful

  2. Offer a contract that requires the agent to be careless

- Note option 2) is equivalent to offering a contract without any requirement (it will be clear later that the agent will choose carelessness if possible)

Maximization with two contract types

- The insurance company's maximization problem will now also depend on which of the two contracts (careful / careless) to choose

- We could write this as one big maximization problem, but it gets pretty extensive and cumbersome; easier procedure:
  1. Suppose the company chooses to require caution, find the optimal contract
  2. Suppose the company chooses to require (permit) carelessness, find the optimal contract
  3. Compare contracts from 1) and 2): The overall optimal contract is the one with the highest expected profit

The cautious contract I

- Let's first check what the company would do if it chose to require the agent to be careful

- The maximization problem here is completely as before except that the risk is now π_f and the agent now incurs a utility cost of e by agreeing

$$\max_{c_1,c_2} \quad M - \pi_f \cdot L - (1-\pi_f)c_1 - \pi_f c_2$$

s.t.

$$\pi_f \cdot v(c_2) + (1-\pi_f) \cdot v(c_1) - e \geq \bar{v} \qquad (\text{IR}_f)$$

The cautious contract II

- As before, we can solve by using Lagrange and the FOCs becomes:

$$\begin{aligned} -(1-\pi_f)+\lambda(1-\pi_f)v'(c_1)=0 &\iff v'(c_1)=\frac{1}{\lambda} \\ -\pi_f+\lambda\pi_f v'(c_2)=0 &\iff v'(c_2)=\frac{1}{\lambda} \end{aligned}$$

- Conclusion: The optimal contract when caution is required gives the agent full insurance ($c_1 = c_2$ i.e. $K = L$)

The careless contract I

- Now let's check what the company would do if it chose to require the agent to be careless
- The maximization problem here is completely as at the beginning except that the risk is now π_s

$$\max_{c_1,c_2} \quad M - \pi_s \cdot L - (1-\pi_s)c_1 - \pi_s c_2$$

$$\text{s.t.}$$

$$\pi_s \cdot v(c_2) + (1-\pi_s) \cdot v(c_1) \geq \bar{v} \qquad (\mathrm{IR}_s)$$

The careless contract II

- Again we can solve by Lagrange and again the FOCs are the same:

$$
\begin{aligned}
-(1-\pi_s)+\lambda(1-\pi_s)v'(c_1)=0 &\iff v'(c_1)=\tfrac{1}{\lambda}\\
-\pi_s+\lambda\pi_s v'(c_2)=0 &\iff v'(c_2)=\tfrac{1}{\lambda}
\end{aligned}
$$

- Conclusion: The optimal contract when carelessness is required also gives the agent full insurance ($c_1 = c_2$ i.e. $K = L$)

- Intuition: No matter whether the agent is careless or not, the risk-neutral Principal shall take all the risk.

Are the contracts so similar?

- No matter if the company requires caution or carelessness, full insurance is offered; BUT the price of the contract is different

- Let Γ_f and Γ_s be the prices: In both cases, $K = L$ (full insurance) and since the constraints bind we use (IR $_f$) and (IR $_s$):

$$\pi_f \cdot v(M-\Gamma_f) + (1-\pi_f)\cdot v(M-\Gamma_f) - e = \bar{v} \iff v(M-\Gamma_f) - e = \bar{v}$$

$$\pi_s \cdot v(M-\Gamma_s) + (1-\pi_s)\cdot v(M-\Gamma_s) = \bar{v} \iff v(M-\Gamma_s) = \bar{v}$$

- v is an increasing function and $e > 0$ so $\Gamma_f < \Gamma_s$: The cautious contract must be cheaper! Intuition: The cautious contract involves a cost to the agent, so the price must be lower for the agent to agree.

Which contract does the insurance company choose?

- The insurance company will compare the profits from offering the contract with caution vs. the careless one and choose to offer the most profitable one.

- Depending on e and π_f vs π_s, either one or the other may be optimal (see Sloth for the math here):
  - The cautious contract lowers expected costs to the insurance company because the risk is lower $\pi_f < \pi_s$ but requires a lower price to be accepted due to higher cost to the agent ($e > 0$)
  - The careless contract increases the expected costs to the insurer, but can be sold at a higher price
  - If e is high and / or the difference between π_f and π_s is small then it is optimal to offer a contract that requires (permits) carelessness

Socrative Quiz Question

True or false: The outside utility of the agent will always be strictly lower if the probability of an accident with caution π_f increases, while all other things (including π_s and e) stay the same.

Amendment 2: Behavior cannot be included in the contract

- Now we change the model again and assume: The insurer cannot observe if the agent is cautious and / or cannot make coverage dependent on the agent's behavior

  (Think about the agent's choice of having a fancy or a discrete bicycle)

- Again, two possible things the insurance company can do:
  1. Offer a contract based on the agent being cautious
  2. Offer a contract based on the agent being careless

- This is a clear case of moral hazard: If the company offers the cautious full insurance contract, the agent will always choose to be careless after signing the contract

Caution IC condition

- First, the contract based on caution now must be designed in a way that makes the agent willing to be cautious; giving the new IC side condition:

$$\pi_f \cdot v(c_2) + (1-\pi_f) \cdot v(c_1) - e \geq \pi_s \cdot v(c_2) + (1-\pi_s) \cdot v(c_1) \qquad (\text{IC}_f)$$

$$(\pi_s - \pi_f) \cdot (v(c_1) - v(c_2)) \geq e \qquad (\text{IC}_f)$$

- Highly intuitive:
  - LHS: cost to the agent arising from being careless: an increased probability of an accident, i.e. of getting c_2 instead of c_1, multiplied by foregone amount of utility in the case of an accident compared to no accident.
  - RHS: effort cost to the agent of applying caution (e).
  - Obviously this cannot be achieved by full insurance (the gain on the left is zero if $c_1 = c_2$); moral hazard!

Cautious contract with moral hazard I

- Principal-agent problem for the cautious contract:

$$\max_{c_1,c_2} \quad M - \pi \cdot L - (1-\pi_f)c_1 - \pi_f c_2$$

$$\text{s.t.}$$

$$\pi_f \cdot v(c_2) + (1-\pi_f) \cdot v(c_1) - e \geq \bar{v} \qquad (\text{IR}_f)$$

$$(\pi_s - \pi_f) \cdot (v(c_1) - v(c_2)) \geq e \qquad (\text{IC}_f)$$

- Can show that (IC_f) must bind: optimal contract from problem *without* (IC_f) had full insurance and did not comply with (IC_f) so at the new optimum (IC_f) must bind.

- IR_f also binds.

Cautious contract with moral hazard II

- Both constraints bind, so we have two equations with the two unknowns c_1 and c_2, and the optimal contract is defined by:

$$\pi_f \cdot v(c_2) + (1 - \pi_f) \cdot v(c_1) - e = \bar{v} \qquad (\text{IR}_f)$$

$$(\pi_s - \pi_f) \cdot (v(c_1) - v(c_2)) = e \qquad (\text{IC}_f)$$

- Solve for $v(c_1)$ and $v(c_2)$ (solve for $v(c_1)$ in the first equation and insert in the second and solve for $v(c_2)$, etc..):

$$v(c_1) = \bar{v} + \frac{\pi_s}{\pi_s - \pi_f} e$$

$$v(c_2) = \bar{v} - \frac{1 - \pi_s}{\pi_s - \pi_f} e$$

Cautious contract with moral hazard III

$$v(c_1) = \bar{v} + \frac{\pi_s}{\pi_s - \pi_f} e$$

$$v(c_2) = \bar{v} - \frac{1 - \pi_s}{\pi_s - \pi_f} e$$

- As long as $e > 0$, the utility without accident will be strictly higher than with accident $v(c_1) > v(c_2)$
- v is an increasing function, which means that $c_1 > c_2$: there is no longer full insurance.
- Now there will be a deductible in the insurance contract! The agent must cover some of the loss himself.

Effect of moral hazard

- Moral hazard entails excess risk: If we want the agent to be careful and we cannot force him to do so he must bear some of the loss himself
- Moral hazard entails an *information cost* for the company, mathematically: The moral hazard has introduced a binding constraint in the profit maximization problem $\Rightarrow$ lower profit
- *Information cost*, intuition: The insurance company cannot design a contract that is as good as before from the agent's point of view, which lowers the agent's willingness to pay.
- The above conclusions are based on going from a cautious contract without moral hazard to moral hazard; remember that the insurance company can in principle also choose the careless contract.

Socrative Quiz Question

True or false: If there is a risk of moral hazard and the cautious contract is selected, this does not only make the company worse off, but also the agent (compared to a cautious contract without the risk of moral hazard).

Careless IC condition

- If the company designs a contract with carelessness, the IC condition is just the opposite of before:

$$\pi_f \cdot v(c_2) + (1-\pi_f) \cdot v(c_1) - e \leq \pi_s \cdot v(c_2) + (1-\pi_s) \cdot v(c_1) \qquad (\mathrm{IC}_s)$$

- Rewrite as before:

$$(\pi_s - \pi_f) \cdot (v(c_1) - v(c_2)) \leq e \qquad (\mathrm{IC}_s)$$

- Note: If we offer full insurance, this is always met (the left side is zero if $c_1 = c_2$

- Intuition (same as before): With full insurance, there is no incentive to be cautious

Careless contract with moral hazard I

- The principal-agent problem for the careless contract:

$$\max_{c_1,c_2} \quad M - \pi \cdot L - (1-\pi_s)c_1 - \pi_s c_2$$

$$\text{s.t.}$$

$$\pi_s \cdot v(c_2) + (1-\pi_s) \cdot v(c_1) \geq \bar{v} \qquad (\text{IR}_s)$$

$$(\pi_s - \pi_f) \cdot (v(c_1) - v(c_2)) \leq e \qquad (\text{IC}_s)$$

Careless contract with moral hazard II

- But it is a very easy problem to solve...:
  - If we removed IC$_s$, we had the same problem as without moral hazard; optimum here had full insurance ($c_1 = c_2$)
  - Under full insurance, IC$_s$ always holds, so the old optimum meets the constraints of the new problem
  - The company can never be made better off by adding a side-condition $\Rightarrow$ the optimal contract is the same as before! (and (IC$_s$) does not bind)
- If the insurance company chooses that the agent should (must) be careless, then it is optimal to offer full insurance again

Which contract does the company choose?

- The insurance company must choose between the careless and the cautious contract
  - The cautious contract lowers expected costs because the risk is lower $\pi_f < \pi_s$, but requires a lower price to be accepted (since $e > 0$) (same as without moral hazard)
  - The cautious contract now requires the introduction of a deductible, which lowers the agent's willingness to pay further, and therefore requires an even lower price for the agent to accept (new, due to the moral hazard problem)
- We could mathematically compare the profits from the two optimal contracts and see which one generates the highest profit; depends on v, e, π_s, π_f and L

Socrative Quiz Question

True or false: If the difference $\pi_s - \pi_f$ becomes very small, it becomes more likely that the careless contract becomes optimal for the company.

The labor market revisited I

- Remember our labor market example in lecture 3a, where we found that the optimal contract was franchising or extreme performance payments

- Claim in slides: If there was uncertainty in production and the agent is risk-averse, the optimal contract would be some kind of profit-sharing:
  - Worker's salary depends to some extent on output, so there is an incentive to work (counteracting moral hazard)
  - However, the worker's salary does not depend too much on the uncertain output (the worker is not exposed to too much risk)

- The intuition (and most of the math) here is exactly the same as in our insurance model

The labor market revisited II

- Slides 3b:
  P. is company, A. is consumer:
  - Cautiousness lowers overall accident costs
  - Caution has a utility cost for A.
  - If the contract is limited by moral hazard, then A. will only be cautious if he is affected by the accident (deductible)
  - Uncertainty about whether an accident (and loss) occurs is bad for A. due to risk aversion.
- Slides 3a with uncertain production:
  P. employer, A. worker:
  - More working hours increase overall production
  - More working hours have a utility cost for A.
  - If the contract is limited by moral hazard, then A. will only work if his salary is affected by the size of the production
  - The uncertainty about the salary is bad for A. because he is risk-averse

What have we learned?

- If there are no moral hazard issues, it is optimal (for everyone) to offer risk-averse consumers full insurance
- Moral hazard problems occur if the agent can affect the risk herself and this behavior cannot be conditioned on in the insurance contract
- With moral hazard issues, it may be necessary to introduce a deductible to ensure that the agent behaves cautiously
- Moral hazard problems can imply lost profit (information costs) for the Principal (insurance company / employer)