Mikro II, lecture 3a

Asymmetric information: Principal-Agent Models

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Plan for the lecture

1. Introduction to Principal-Agent models
2. The optimal solution (contract) in a simple Principal-Agent model
3. Optimal solutions (contracts) in a simple Principal-Agent model with moral hazard
4. Formalizing the Principal-Agent model with moral hazard

Asymmetric information

- The next source of market failure is about information

- When the first welfare theorem is described it is often mentioned that “perfect information” is assumed

- But in Micro I you have already seen imperfect information: Decisions under uncertainty

- Now we will examine some problems that arise when we have imperfect information:
  - Someone in the market knows more than others...
  - ... and it is information that is directly relevant to the decisions of others (pendant to externalities)

Principal-Agent Models I

- Before we move on to the topics of moral hazard and adverse selection that can arise from asymmetric information, we need a little methodological detour

- We need to look at *Principal-Agent* models (or Principal-Agent theory / problems), which are central to *contract theory*.

- Often economic behavior is about designing and agreeing on a "contract" rather than just trading a fixed item and agreeing on a price:
  - Insurance: How much is covered? Under what conditions?
  - Labor market: How many regular hours per week? Overtime? Fixed salary or bonuses?
  - Loan market: When and how to repay? What happens if you cannot repay?

Principal-Agent Models II

- *Contract Theory* is a major, important field in theoretical microeconomics (e.g. Nobel Prize to Oliver Hart and Bengt Holmström in 2016)

- Principal-Agent models in a nutshell:

1. A principal (employer, insurance company, bank) must design and offer a contract to an agent (worker, consumer, entrepreneur)
2. The principal wants to maximize his/her own profit (utility), but must take into account that the agent must say yes (the agent's incentives)
3. The principal faces some limitations: asymmetric information about the agent type, or the agent can do something the principal does not want

Example

- There is an employer who must offer a contract to a worker; we call the employer the principal and the worker the agent
- The principal will maximize profits, which depend on how much the worker works and the salary that is paid to him
- The contract is a take-it-or-leave-it offer to the Agent: the worker compares the utility of the contract with his reservation benefit (outside option)
- The principal must therefore take into account that the worker must get utility of at least his reservation utility

The principal's problem

- If we rewrite the previous slide semi-mathematically we get:

$$\max_{\text{contract}} \quad \text{profit}$$

s.t.:

worker's utility from contract $\geq$ reservation utility (outside option)

The principal's problem

- Work time is e (effort), wage is w and production is a function of the work time $y = f(e)$, where $f(0) = 0, f' > 0, f'' < 0$
- The worker has utility function $u(w, e)$, increasing in wage (w), decreasing in work time e and reservation utility $\bar{u}$ (for example benefit from unemployment benefit)

$$\max_{w,e} \quad y - w$$

$$\text{s. t.}$$

$$u(w, e) \geq \bar{u} \qquad \text{(IR)}$$

- The condition that it must be rational to say yes is called *Individual Rationality* constraint (IR) or sometimes *Participation Constraint*

Solution I

- Assume the simple utility function $u(w,e) = w - e$ and let us solve; the constraint must be binding (otherwise we could increase profits by lowering the wage) so we can solve for w:

$$u(w,e) = \bar{u} \iff w - e = \bar{u} \iff w = \bar{u} + e$$

- Insert in the maximization problem (now depends only on e) and use that $y = f(e)$:

$$\max_{w,e} \quad f(e) - (\bar{u} + e)$$

- First order condition yields (let e^*, w^* be the solution):

$$f'(e^*) - 1 = 0 \iff f'(e^*) = 1$$

Solution II

- First order condition yields (let e^*, w^* determine the solution):

$$f'(e^*) - 1 = 0 \iff f'(e^*) = 1 \iff e^* = (f')^{-1}(1)$$

- The equations determine the work time, the constraint yields:

$$w^* - e^* = \bar{u} \iff w^* = \bar{u} + e^*$$

- Intuition (rather boring): The optimal contract ...
  - ... sets the working time such that the marginal product is equal to the worker's marginal disutility (in dollars) ...
  - ... sets the wage to precisely match the reservation benefit plus the disutility of work, so the constraint is satisfied and the worker agrees to the offered contract and the company benefits (optimally)

Socrative Quiz Question

True or false? If f is linear in e, there will be infinite production.

Efficiency

- As always, we might be interested in asking if the contract is socially efficient
- Intuitively it works efficiently and it is actually very quick to show that it *must* be:
  1. Principal-Agent problem puts the company as well off as possible, provided the agent has at least utility $\bar{u}$
  2. By definition, we can therefore not make the company better off without making the agent worse off
  3. Suppose we could do something to make the agent better off without making the company worse off, then we could lower w or increase e and thus increase profits $\Longrightarrow$ contradicts that the contract is a solution
  4. Add 2) and 3) together: The contract that solves the Principal-Agent problem is Pareto-optimal (i e efficient)!

Obstacles

- As mentioned initially, Principal-Agent models only get really interesting when we insert several (realistic) obstacles for the Principal

- A classic obstacle: The Principal cannot enforce the contract perfectly:
  - Before, we assumed that the Principal could offer a contract with a specific working time and be sure it was followed if the agent said yes

  - In practice, it may be difficult to check (or enforce) whether the agent is working as much as the contract requires

  - New assumption: After the agent has agreed to the contract from before, the agent can freely choose to work less (or more)

The contract from before

- We return to the Principal's formal maximization problem with obstacles later, but let's first check what would happen if we just re-used the contract from before:
  - The agents agrees to the contract and receives the wage $w^* = \bar{u} + e^*$
  - Now the agent must choose how much he/she wants to work:

$$\max_{e} \quad w^* - e$$

  - Easy solution: w^* is a constant so work as little as possible: $e = 0$ (technically a corner solution , we assume implicitly that $e \geq 0$)
  - Very unfortunate contract! The Principal pays wages but gets nothing in return; example of *Moral Hazard*; the agent's *misbehavior* after saying yes makes the contract fail.

Other contracts

- Now let's consider some other "contracts" that might work better
- Remember that the optimal outcome without obstacles was $e = e^* = (f')^{-1}(1)$. Can we implement this? (remember this is efficient)
- Procedure: Try some specific "contracts" and see what they result in (we'll link them to the maximization problem later)
- Despite the formal maximization problem we have set up, it is often good practice to just "guess and check" a specific contract in this way to better understand the intuition.

Franchising I

- A form of contract that you see in real life is “franchising”: You pay me a fixed amount so I allow you to run a production, and in return you can keep the profits yourself as a salary.

- The typical example is, historically, McDonald’s managers: rather than hiring a manager to run the local McD, one uses franchises

  (other prominent Danish examples: Rema 1000, 7Eleven)

- In the context of our Principal-Agent problem:
  - The worker pays (commits to pay) a fixed amount R to the principal, but then has to choose his own working hours and keep the production $y = f(e)$

Franchising II

- When the worker has to choose his working time, he now maximizes:

$$\max_e \quad u(f(e)-R,e)=\max_e \quad f(e)-R-e$$

- The first order condition yields $e^* = (f')^{-1}(1)$ so franchising implements the efficient solution.

- Intuition: The worker gets the full marginal return on his working time and therefore chooses to work optimally (the worker is *residual claimant*)

- Check yourselves at home: What is the maximum willingness to pay of the worker for the franchise? (think of the worker's reservation utility)

Socrative Quiz Question

If the Principal designs the franchise contract that is offered to the worker optimally, the principal's profits will . . .

- a) . . . be higher than without asymmetric information.
- b) . . . be the same as without asymmetric information.
- c) . . . will be lower than without asymmetric information but will not be zero.
- d) . . . will be zero.

Performance-related pay policies I

- Another contract used in practice is different types of performance pay, where the salary depends on how the company is doing.

- Consider the following (extreme) version: Your salary is $w^* = \bar{u} + e^*$ if you sell at least $f(e^*)$ and otherwise it is 0:

- Note: the agent can make sure that the wage is $w^* = \bar{u} + e^*$ by working at least e^*

- Note: obvious that an agent with this contract will either decide not to work, i e $e = 0$, or will decide to work exactly enough to make the wage non-negative, i e $e = e^*$ (extra work yields nothing).

Performance-related pay policies II

- Saying yes to the contract but then not working (i e no salary) yields a benefit of 0; working $e = e^*$ yields the utility $w^* - e^* = \bar{u}$
- The agent is thus clearly willing to work $e = e^*$ under the performance related pay policy
- The contract is thus another way of restoring the efficient outcome of the old contract
- The contract gives the agent incentive to work as much as the principal would like

Profit sharing I

- Finally, we can consider some form of profit sharing, where the agent gets a base salary $\bar{w}$ plus a share $a < 1$ of production (or profit)

$$w = \bar{w} + af(e)$$

- The agent will maximize:

$$\max_{e} \quad w - e = \max_{e} \quad \bar{w} + af(e) - e$$

- First order condition is (let $\hat{e}$ be the solution): $af'(\hat{e}) = 1$

Profit sharing II

- The first order condition is (let $\hat{e}$ be the solution): $af'(\hat{e}) = 1$;

$$\hat{e} = (f')^{-1}\left(\frac{1}{a}\right)$$

- You can show that $\hat{e} < e^* = (f')^{-1}(1)$ when $a < 1$ ($f'' < 0 \Rightarrow f'$ decreasing $\Rightarrow (f')^{-1}$ decreasing; also note that $\frac{1}{a} > 1$ when $a < 1$)

- Thus, profit sharing does not restore efficiency; intuition: the agent receives only part of the production they make (i e part of the marginal benefit of work), so he ends up working too little.

Socrative Quiz Question

True or false? If the principal designs the contracts optimally, the worker will always prefer profit-sharing to franchising.

Summing up

- When it is not possible to enforce a specific working time, then the following applies:
  - Franchising can restore the socially efficient outcome of the old contract.
  - Performance-related pay policies (in fairly extreme form) can restore the socially efficient outcome of the old contract.
  - Profit sharing does not provide efficiency.

(We will talk later about why a more realistic model significantly changes the above conclusions)

Back to the maximization problem

- We started with a very formal maximization problem for the principal, assuming that the contract could force the agent to a specific working time
- Then we introduced moral hazard by removing the last assumption and then followed up with something slightly unsystematic *”if the contract has these specific characteristics then ... ”* - analysis
- Now let’s jump back and modify the maximization problem to formalize our assumption of non-enforceability of working time (moral hazard)

The principal's problem

- Remember our semi-mathematical formulation of the maximization problem (now including the (IR) label and using the ”s.t.” abbreviation):

$$\max_{\text{contract}} \quad \text{profit}$$

$$\text{s.t.}$$

$$\text{worker's utility} \geq \text{reservation utility (outside option)} \qquad \text{(IR)}$$

The new problem

- Modify the problem so we require the worker to both say yes and end up choosing the "right" amount of working time:

$$\max_{\text{contract}} \quad \text{profit}$$

(s.t.):

worker's utility ≥ reservation utility (outside option) (IR)

worker's utility from working number of hours specified in the contract ≥
the worker's utility from not working according to the contract (IC)

- The new side condition is called the *Incentive compatibility* constraint (IC) (the agent must have an incentive to comply with the contract's working hours)

Naïve Mathematics (NB LITTLE WRONG)

$$\max_{w,e} \quad y - w$$

s. t.

$$u(w,e) \geq \bar{u} \qquad \text{(IR)}$$

$$u(w,e) \geq u(w,e') \quad \text{for all} \quad e' \qquad \text{(IC)}$$

Interpretation of (IC): There must be no other working time e' which the agent likes better than the one specified in the contract (which he will therefore choose instead)

What was wrong?

- Previous slides made it clear how to write (IC) as a mathematical condition, but there was something wrong

- The problem in the previous slide was to select a working time e and a salary w

- But if you think back to our performance wage example then the whole point was that it was a contract where the salary depended on what the agent did

- The contract we choose should not specify a fixed salary level, but must specify what the salary is as a function of what the agent does

What can the salary depend on?

- One possible way that wages may depend on the agent's behavior: Wages depend directly on working hours e, that is $w(e)$
  - In principle just fine, but we just questioned whether the company could actually check how much the agent worked

- A better way (here) is that we only allow contracts where the salary depends on output y, i e $w(y)$
  - Interpretation: The company always observes how much it produces and can set the salary accordingly even if it does not see working hours directly
  - Note: The distinction between $w(e)$ and $w(y)$ is slightly artificial right now because output and working time are related completely one-to-one ($y = f(e)$), but wait a bit!

The principal's problem I

- Now we can proceed by recognizing that the contract specifies how wage and output are related (remember $y = f(e)$):

$$\max_{w(\cdot),e} \quad y - w(y)$$

$$\text{s. t.}$$

$$u(w(y), e) \geq \bar{u} \qquad \text{(IR)}$$

$$u(w(y), e) \geq u(w(y), e') \quad \text{for alle} \quad e' \qquad \text{(IC)}$$

- A maximization problem where to find the optimal *function* is reasonably heavy math ...

The principal's problem II

- ... but you can go a long way with a *"if the contract has these specific features ..."* analysis

- It can be shown that our performance-pay contract and franchising formally yield two different solutions to the problem (with different $w(\cdot)$ function); Sketch of the proof:
  1. w^*, e^* was a solution to the original problem that did not have the (IC) condition
  2. Performance payment contract and franchise implement w^*, e^* even with the (IC) condition
  3. The principal can't be made better off by adding a constraint, so w^*, e^* must be the optimum with asymmetric information too.

Loose ends

- Why should the principal choose a specific working time e *and* a payment function $w(y)$; is e not given once the agent faces $w(y)$?

  Well, basically yes. However, it is mathematically useful to maximize with respect to e

- Couldn't we imagine a situation where the optimal outcome obtains, with a contract that wasn't perfectly complied with, but where the agent did something a little different than intended? Well, but in that case we can redefine the working time requirement of the old contract to match the actual behavior of the agent

Summing up

- We have now (very thoroughly) set up a classic Principal-Agent problem between an employer and a worker with a moral hazard problem

- In the Principal-Agent problem, the employer must design the best possible contract, taking into account the worker's incentives (outside option and risk of moral hazard)

- We have seen that two optimal solutions to the moral hazard problem were a franchising contract and a fairly extreme performance pay contract, while profit sharing (or less extreme performance pay schemes) did not work

Socrative Quiz Question

True or false? In principal agent problem we have just seen, asymmetric information has no welfare cost.

Compare to reality

- In practice, there is not *so* many places in the labor market where we see franchising or performance pay this extreme.
    - Can be that there may not be a moral hazard problem (employers can perfectly monitor employees) ...
    - ... or that employees / employers do not exhibit optimizing behavior at all in practice
- But it may also be due to something we did not include in the model here: Franchising and performance payments result in great *uncertainty* for the worker if production (sales) can fail

Uncertainty (preview at Sloth note)

- Imagine that we changed the model such that the production involved some uncertainty (luck) no matter how much the employee works

- Also, imagine that we assume the worker is risk-averse

- Then, extreme performance payments and franchising would no longer be optimal: the worker would require very good terms to accept so much risk

- In this case, the optimal contract would instead be a version of profit sharing:
  - The worker's salary depends to some extent on output so there is an incentive to work (counteracting moral hazard)
  - However, the worker's salary does not depend too much on the uncertain output (the worker is not exposed to too much risk)

Socrative Quiz Question

True or false? If both the principal and the agent are risk-averse then the market will break down and there is zero production.

What have we learned?

- What a Principal-Agent model is
- What an Individual Rationality constraint covers
- An example of moral hazard in a Principal-Agent model
- What an Incentive Compatibility constraint is (in a situation with moral hazard)