UNIVERSITY OF COPENHAGEN

# Mikro II, lecture 2b

## Externalities, property rights and Coase

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# Plan for the lecture

1. Tragedy of the Commons
2. Coase theorem, first part
3. Externalities in the Edgeworth box
4. Coase theorem, second part

# An exaggerated externality tragedy

- Old school agricultural method (which is still used today):
- A village is surrounded by a common resource (fields) where the sheep are grassing
- Now: A mathematical analysis of the amount of sheep grassing

# The farmers and the sheep

- There are $N$ farmers who decide how many sheep to put out
- Let $x_1$ be the number of sheep that farmer $1$ puts out and $x_2$, $x_3$ etc. be the amount of sheep that the other farmers put out
- The total number of sheep is $X = x_1 + x_2 + \ldots + x_N$

# Sheep technology

- There is a cost $a$ of buying a sheep that can grass on the fields
- The return of having a sheep grass is $f(X)$ that is, it depends on the number of sheep that other farmers put out
- The more sheep grass, the less grass there is left so $f$ is decreasing ($f' < 0$); also assume that f is concave ($f'' < 0$)

# The social optimum

- How many sheep will we have to put out to maximize total profits?

$$\max_{X} \quad X \cdot f(X) - a \cdot X$$

- The first order condition determines the solution to this problem (f is concave):

$$f(X^{opt}) + X^{opt} \cdot f'(X^{opt}) - a = 0 \quad \Longleftrightarrow$$

$$\underbrace{f(X^{opt})}_{\text{Return of another sheep}} = \underbrace{a}_{\text{The price for a sheep}} - \underbrace{X^{opt} \cdot f'(X^{opt})}_{\text{One more sheep decreases the return on all other sheep}}$$

# The farmers' problem I

- Farmer 1 will choose the amount of sheep that maximizes his private profit:

$$\max_{x_1} \quad x_1 \cdot f(X) - a \cdot x_1$$

- First order condition determines the solution (remember that $X = x_1 + x_2 + \ldots + x_N$):

$$f(X) + x_1 \cdot f'(X) - a = 0 \iff f(X) = a - x_1 \cdot f'(X) \iff x_1 = \frac{a - f(X)}{f'(X)}$$

- (note the implicit assumption here: Farmer 1 is taking the amount of sheep that other farmers put out as given)

# The farmers' problem II

- The other farmers solve the exact same problem so for every farmer $i$ we have:

$$x_i = \frac{a - f(X)}{f'(X)}$$

- By adding together all the first order conditions we get an equation that determines the amount of sheep that will be put out in equilibrium, $X^B$:

$$x_1 + x_2 + \ldots x_N = \frac{a - f(X^B)}{f'(X^B)} + \frac{a - f(X^B)}{f'(X^B)} \ldots + \frac{a - f(X^B)}{f'(X^B)} \iff$$

$$X^B = N \cdot \frac{a - f(X^B)}{f'(X^B)} \iff$$

$$f(X^B) = a - \frac{X^B}{N} f'(X^B)$$

# Comparison to the social optimum

- Compare the two sets of equations:

$$\underbrace{f(X^{opt})}_{\text{Return from another sheep}} = \underbrace{a}_{\text{The price of a sheep}} - \underbrace{X^{opt} \cdot f'(X^{opt})}_{\text{An extra sheep lowers the return from all } X^{opt} \text{ sheep}}$$

$$\underbrace{f(X^{B})}_{\text{Return from another sheep}} = \underbrace{a}_{\text{The price of a sheep}} - \underbrace{\frac{X^B}{N} \cdot f'(X^{B})}_{\text{An extra sheep lowers the return on } \frac{X^B}{N} \text{ sheep???}}$$

- The decision of the farmers is the same as maximizing total profits (of all farmers) except that you *behave as if* an extra sheep only lowers the return of an $N$'th fraction of the sheep

# Externality and inefficiency

- Every farmer only owns an $N$'th fraction of the total amount of sheep and will hence only take into account the effect on his own sheep on the amount of grass
- Externality: The farmers do not take into account how an extra sheep affects other farmers' sheep
- Inefficiency: Intuitively there will be too many sheep when all farmers make their decision in this way ($X^B > X^{opt}$)
- Tragedy of the Commons! Individual decision-making leads to over-consumption.

# Math behind the tragedy of the commons I

- Look at the equation for the number of sheep when the farmers make their decisions individually:

$$f(X^B) + \frac{X^B}{N} f'(X^B) - a = 0$$

- This function will define $X^B$ implicitly as a function of $N$: $X^B(N)$

- By implicit differentiation we can analyse $\frac{dX^B}{dN}$, let $LHS$ determine the left-hand side of the equation:
  - How does the left hand side change with $X^B$? Differentiate: $\frac{dLHS}{dX^B} = f'(X^B) + \frac{1}{N} f'(X^B) + \frac{X^B}{N} f''(X^B) < 0$ so left hand side decreases in $X^B$
  - How does the left hand side change with $N$? Differentiate: $\frac{dLHS}{dN} = -\frac{X^B}{N^2} f'(X^B) > 0$ so left hand side increases in $N$

# Math behind the tragedy of the commons II

$$f(X^B) + \frac{X^B}{N} f'(X^B) - a = 0$$

- Left hand side is decreasing in $X^B$ and increasing in $N$
- What happens if $N$ goes up?
  1. The left hand side increases and will therefore become $> 0$
  2. In order for the equation to hold $X^B$ will have to change such that the left hand side decreases again
  3. So $X^B$ will have to increase
- Conclusion: $\frac{dX^B}{dN} > 0$

# Math behind the tragedy of the commons III

- Partial conclusion 1: The number of sheep increases with the number of farmers.

- Note that if $N = 1$ the equation giving the number of sheep when the farmers decide for themselves is equal to the socially optimal number of sheep $X^{opt}$.

- Partial conclusion 2: The socially optimal number of sheep will result if there is only a single farmer.

- Partial conclusions 1 and 2 together prove the over-consumption results: If $N > 1$ then $X^{opt} < X^B$.

# From sheep to fish and cars

- The idea behind the “Tragedy of the Commons” is a central problem in economics

- It is not because there is a specific interest in sheep:
  - Tragedy of the commons can explain the inefficiency stemming from over-fishing in the oceans
  - It can also explain the inefficiency stemming from traffic congestion
  - (Find more examples yourselves!)

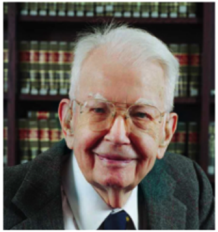
# Externalities and property rights

- We have seen that the externality problem Tragedy of the Commons can be solved by assigning property rights
- We have previously studied schemes that solve the problem as well:
  - Tradable $CO^2$ quotas restore efficiency by assigning the right to emit

# Socrative quiz question

True or false? The tragedy of the commons in the model we studied cannot be remedied by imposing a unit tax t on the purchase of a sheep.

# Coase theorem



- All this is an example of the "Coase Theorem" (Ronald Coase, Nobel prize in 1991)

- (Note that here (and often) the Coase theorem is not formulated mathematically)

- Coase Theorem (first part, the important part):

  *If all property rights are assigned, the end result will be efficient (pareto-optimal), no matter how the property rights are assigned*

- Note: This is only the case if transaction costs are small enough

# Nechyba's pool-example

- My neighbor wants to build an extra floor in his house; problem: the extra floor will cover the sunlight for my swimming pool
- "Property rights" are unclear here: does my neighbor have the right to do this? Can I prevent him from doing it?
- We let the court decide.
- Let us assume that the judge is an economist and he only cares about efficiency. What should he decide?

# Nechyba's pool-example

- It is efficient to build the extra floor if my neighbor's willingness to pay for the floor is higher that my willingness to pay for sunlight
- Coase theorem: It does not matter what the judge decides!
- As long as the right to build the floor (or to not having it built) is assigned, the outcome will become efficient

# Coase theorem in action I

- Assume that my willingness to pay is 5,000 DKK and my neighbors willingness to pay is 10,000 DKK

- If the judge decides that my neighbor can build the extra floor:
  - The neighbor builds the floor; this is efficient!

- If the judge decides that my neighbor cannot build the extra floor:
  - My neighbor can offer me somewhere between 5,000 DKK and 10,000 DKK for my permission to build the floor anyway

  - I agree; this is efficient!

# Coase theorem in action II

- Assume that my willingness to pay is 10,000 DKK and my neightbor's willingness to pay is 5,000 DKK

- If the judge decides that the floor can be built:
  - I can offer my neighbor an amount between 5,000 DKK and 10,000 DKK if he does not build the floor
  - The neighbour agrees NOT to build the floor; this is efficient!

- If the judge decides that the floor cannot be built:
  - The neighbor will not build the floor; this is efficient!

# Coase theorem in action III

- No matter who is assigned the property right, the outcome will be efficient (note that the judge's decision will affect the income distribution!)
  - In practice there will be transaction costs, which depend (among others) on the number of people involved.

# Edgeworth-box revisited I

- Two agents: $A$ and $B$; Two "goods": consumption $c_A, c_B \geq 0$ and the fraction of air filled with cigarette smoke $r \in [0,1]$

- Utility functions $u_A(c_A, r)$ and $u_B(c_B, r)$; smoke is in both functions = externality

- Both agents like consumption (as always): $\frac{\partial u_A}{\partial c_A}, \frac{\partial u_B}{\partial c_B} > 0$

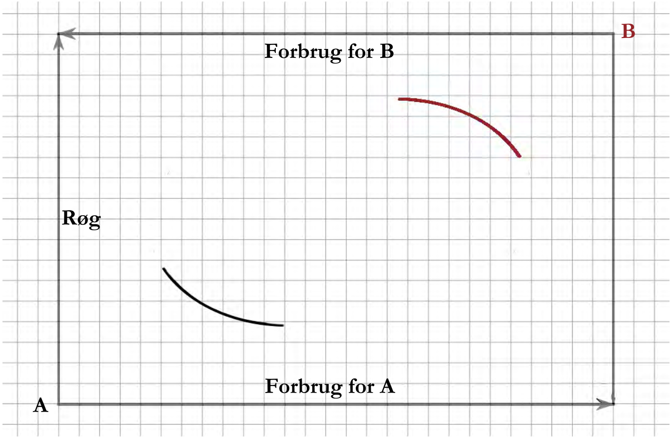
- B is a non-smoker and does not like smoke; A is a smoker and LOVES smoke: $\frac{\partial u_A}{\partial r} > 0, \frac{\partial u_B}{\partial r} < 0$

- Initial endowment of resources (consumption): $(\bar{c_A}, \bar{c_B})$, $C = \bar{c_A} + \bar{c_B}$

# Edgeworth-box revisited I

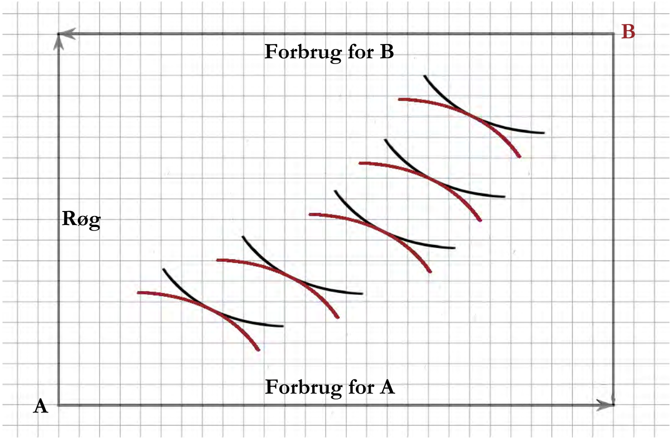
- This small economy is depicted in the modified Edgeworth box:
  - As before: The $x$-axis indicates the distribution of consumption between the agents
  - The new thing: The $y$-axis indicates how much smoke both agents observe
  - A likes consumption and smoke so he favors the upper-right part of the box (as before)
  - B likes consumption but dislikes smoke so he favors the lower-left part of the box (also like before)

# Edgeworth-box revisited II



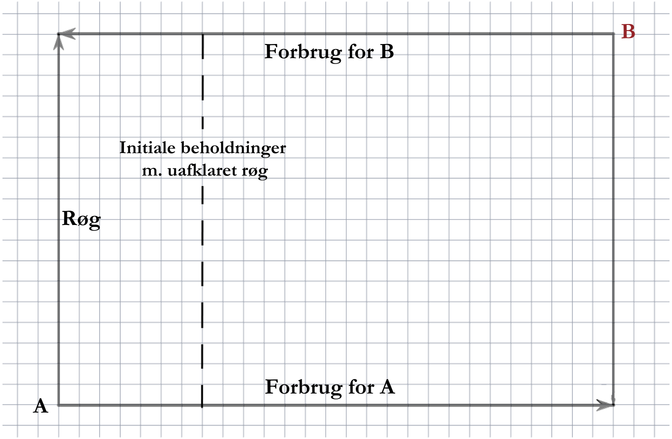

- We can now do the same analysis/exercises as we did in Micro I

# The contract curve (efficient allocations)



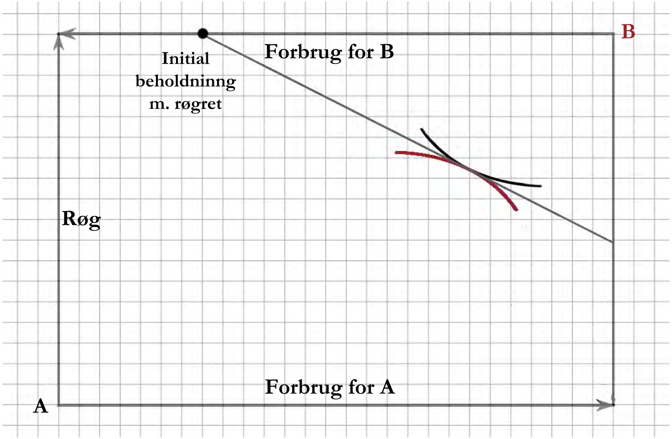

- The optimal (efficient) allocation is determined by the tangency between A and B's indifference curves

# Initial endowment? The right to smoke?




- Assume that the initial endowment will be somewhere on the line above.

# Coase gives an efficient equilibrium amount of smoke




- Coase: If property rights are assigned and trading is allowed, we end up with an efficient equilibrium

# Summing up

- The modified Edgeworth box illustrates the Coase theorem
- If we assign property rights over the externality and allow trading to take place, we are back to the Walras equilibrium from Micro I.
- The first welfare theorem holds for this economy, so the equilibrium is efficient.

# Coase theorem second part

- The first part of the Coase theorem says that we get efficiency no matter how property rights are assigned

- The distribution of property rights will matter for the final distribution of resources

- The second part of the Coase theorem points out a special case where the distribution does not matter for the equilibrium quantity of the externality:

  *If the agents have quasi-linear preferences, the equilibrium quantity of the externality is independent of how the property rights are distributed*

  (Thus: we end up with the same amount of $CO_2$ / smoke no matter how the rights are distributed)

# Coase theorem second part, intuition

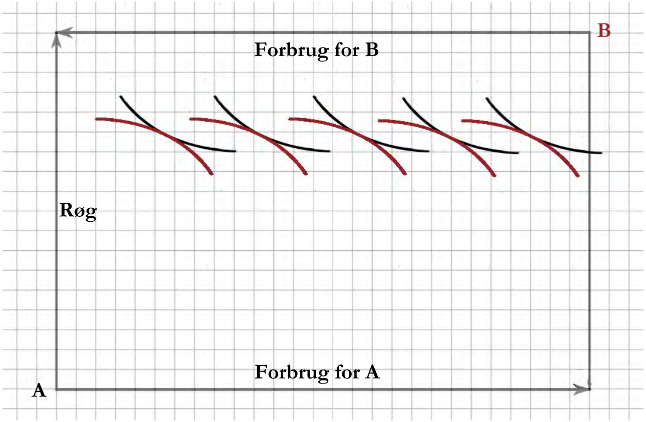
- When we assign property rights, we change people's (potential) income (I get an extra right that I can potentially sell)

- Changed income can change people's willingness to pay (income effects) and the outcome, for example:

  If my neighbor gets the right to build his floor, he feels richer and builds, if not he feels poorer and does not build.

- Coase part 2: Quasi-linear preferences $\Rightarrow$ No income effects $\Rightarrow$ Willingness to pay is independent of the level of income

# Coase second part, figure




- Quasi-linear preferences $\Rightarrow$ Indifference curves are pushed upwards in parallel when the initial endowment changes $\Rightarrow$ The contract curve is a horizontal line and the equilibrium amount of smoke is constant

# Socrative quiz question

True or false? If smoking and consumption of other goods are perfect complements for A, the contract curve will be linear.

# What have we learned?

- Tragedy of the Commons
- Externalities can be seen as a lack of property rights
- Coase theorem first and second part
- Calculate and analyze externalities in the modified Edgeworth box