Mikro II, lecture 2a

Externalities, taxes and quotas

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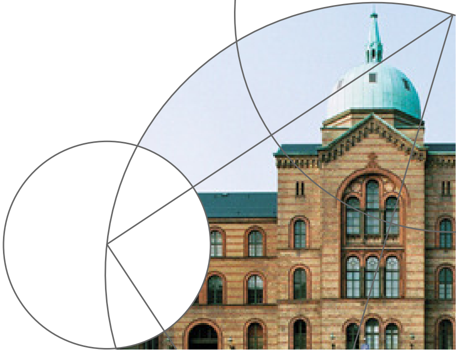


Plan for this lecture

1. A model to illustrate problems with externalities
2. Nechyba's model for production externalities
3. Pigouvian taxes and tradeable quotas in models with externalities

Externalities

- An example of what can go wrong in a competitive market: Externalities

- Think back to Micro I
  - Agent's and firm's utility and profit only derives from own consumption and production.

  - The first and second welfare theorem showed that the market equilibrium was efficient. But do those theorems always hold?

Socrative Quiz Question

What does the second welfare theorem tell us?

- a) The equilibrium in a competitive market without externalities will always be pareto-efficient.
- b) The government will always be able to achieve a Pareto-improvement relative to the market equilibrium through adequate taxation of goods.
- c) Any pareto-efficient allocation can be sustained as an equilibrium in a competitive market without externalities by allocating the initial endowments to the agents in the right way.

Intro: A steel and fish economy

- A small economy with two firms:
  - A steel producing firm that produces and exports steel on the world market; steel production pollutes the ocean/lakes
  - A fishing firm that produces and exports fish to the world market; population of fish depends on pollution
  - A given number of agents that share the combined total profits of the two firms.
- Attention: Easy to think about efficiency in this economy: the higher total profits, the better these agents are made off

Plan of the analysis

- Let us (as always) assume that the two firms maximize their own profits
  - The steel firm chooses how much steel to produce given the technology and prices available. The firm will also choose how much to pollute.
  - The fishing firm chooses how many fish to catch taking technology, prices and pollution as given.
- First we will derive the market equilibrium...
- ... then we will derive the social optimum.

Production technology: Steel

- The steel firm chooses how much steel s to produce and sell at the market price p_s.
- The steel firm pollutes; the level of pollution x depends on how it chooses to produce the steel.
- Technology: The cost function depends on the amount of steel and the amount of pollution: $c_s(s, x)$
- Assumption: By allowing some pollution, the firm can lower production costs.

Profit maximization: Steel

$$\max_{s,x} = p_s \cdot s - c_s(s,x)$$

- First order condition for interior solution (let s^* and x^* determine the solution):

$$p_s = \frac{\partial}{\partial s} c_s(s^*,x^*) \quad , \quad \frac{\partial}{\partial x} c_s(s^*,x^*) = 0$$

- Interpretation (from right to left):
  - Increase the level of pollution until it no longer decreases production costs (given the level of steel production)
  - Increase steel production until the marginal cost are equal to the market price (given pollution)

Profit maximization: Fish

- The fishing firm will choose how many fish f to catch and sell the fish at the market price p_f

- The fish population depends negatively on the level of pollution x, hence pollution increases the cost of catching fish

- Technology: The costs of producing fish depend on the amount of fish and the pollution level: $c_f(f, x)$

Profit maximization: Fish

$$\max_{f} = p_f \cdot f - c_f(f, x^*)$$

- First order condition for interior solution (let f^* determine the solution):

$$p_f = \frac{\partial}{\partial f} c_f(f^*, x^*)$$

- Interpretation:
  - Produce fish up until the point where the price equals marginal costs (given the pollution level)

The social optimum I

- We have now derived the equations that characterize the market equilibrium.

- We would like to compare this equilibrium to the social optimum

- Social optimum is easy to define/find: Choose the amounts of steel production, pollution and fish production that maximize total profits of the two firms:

$$\max_{s,x,f} = p_s \cdot s - c_s(s,x) + p_f \cdot f - c_f(f,x)$$

The social optimum II

$$\max_{s,x,f} = p_s \cdot s - c_s(s,x) + p_f \cdot f - c_f(f,x)$$

- First order condition for interior solution (let s', f', x' be the solution):

$$p_s = \frac{\partial}{\partial s} c_s(s',x')$$

$$p_f = \frac{\partial}{\partial f} c_f(f',x')$$

$$\frac{\partial}{\partial x} c_s(s',x') = -\frac{\partial}{\partial x} c_f(s',x')$$

Compare the FOCs

$$p_s = \frac{\partial}{\partial s} c_s(s', x')$$
$$p_f = \frac{\partial}{\partial f} c_f(f', x')$$
$$\frac{\partial}{\partial x} c_s(s', x') = -\frac{\partial}{\partial x} c_f(s', x')$$

$$p_s = \frac{\partial}{\partial s} c_s(s^*, x^*)$$
$$p_f = \frac{\partial}{\partial f} c_f(f^*, x'^*)$$
$$\frac{\partial}{\partial x} c_s(s^*, x^*) = 0$$

- In both scenarios, steel and fish production determined by setting marginal costs equal to market prices, BUT level of pollution determined differently:
  - Market equilibrium: The steel firm will pollute up until the point where it no longer lowers its (private) costs
  - Social optimum: The level of pollution is increased up until the point where the cost savings on steel equals the cost increment in fish

The externality creates inefficiency

- The market equilibrium is NOT socially optimal because of the externality: The steel firm does not take the damage on the fishery caused by pollution into account when it determines the amount of pollution.

- The market equilibrium makes the agents in our small economy worse off (lower profit $\longrightarrow$ lower income) compared to the social optimum.

- Given reasonable assumption on c_s and c_f (see for example next slide):
  - The level of pollution is too high in the market equilibrium ($x^* > x'$).
  - The market produces too much steel and too few fish ($s^* > s'$ and $f^* < f'$) because a higher pollution level makes it cheaper to produce steel.

Example: Functional forms: (do the calculations yourselves)

- Assume the cost functions: $c_s(s,x) = s^2 + (s-x)^2$ and $c_f(f,x) = f^2 + f \cdot x$
- Assume the market prices obey $p_s > p_f > \frac{1}{2}p_s$ (to exclude corner solutions)
- The market equilibrium becomes: $s^* = \frac{1}{2}p_s$, $x^* = \frac{1}{2}p_s$, $f^* = \frac{1}{2}p_f - \frac{1}{4}p_s$
- Social optimum is: $s' = \frac{3}{4}p_s - \frac{1}{2}p_f$, $x' = p_s - p_f$, $f' = p_f - \frac{1}{2}p_s$

Example: with numbers

- Assume cost functions are the same as before.
- Assume that $p_s = 12$, $p_f = 8$.
- The market equilibrium becomes $s^* = 6$, $x^* = 6$, $f^* = 1$
  fish and steel profits become 1 and 36.
- Social optimum is: $s' = 5$, $x' = 4$, $f' = 2$
  fish and steel profits become 4 and 34
  (and we have a higher total profit compared to the market equilibrium)

Socrative Quiz Question

Assume that pollution makes it easier to produce fish, but everything else is unchanged. How would the profit of the fishing firm compare between the market equilibrium and the social optimum?

- a) Higher in the market equilibrium than in the social optimum.
- b) Higher in the social optimum than in the market equilibrium.
- c) The same in the market equilibrium as in the social optimum.

Fish/steel example: summing up

- This example shows that externalities create inefficient market equilibria
- Natural question: Can we restore efficiency with policy measures?
  - Yes! There are different solutions to solve the problem of externalities
  - There is a very simple solution in our example: Merger of the two firms
    - The profit maximization problem if the two firms become one is the same as for the social optimum (this internalizes the externality)
  - More on other policy measures to restore efficiency in a few slides.

Nechyba's externality model

- Partial equilibrium analysis on a market with perfect competition
- Demand curve (linear): $D(p) = x_d(p) = \frac{A-p}{\alpha}$
- Supply curve (linear): $S(p) = x_s(p) = \frac{B+p}{\beta}$
- We will need to keep track of the firm's marginal costs:
  - Remember (from Micro I) that the inverse supply curve is equal to the marginal cost curve

$$p_s(x) = MC_s(x) = -B + \beta x$$

The externality

- For every unit produced, δ units of CO^2 are emitted
- CO^2 causes society a cost that is not accounted for by the firms (not internalized):
  - With a production of x, there is an external cost of CO^2: $C_E(x) = (\delta x)^2$
  - The marginal CO^2 cost is: $MC_E(x) = 2\delta^2 x$
- The *social* marginal cost curve is: $SMC(x) = MC_s + MC_E(x) = -B + (\beta + 2\delta^2)x$

Interpretation and relation to fish/steel

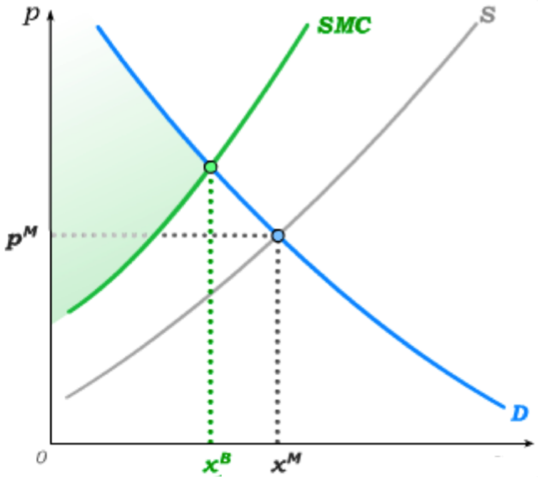
- The idea is the same as before:

  There is a social externality cost that the firms do not think about when choosing their production level

- Nechyba's model is (in a way) more partial than our previous example:
  - Nechyba assumes that social costs are given
  - The fish/steel example showed how this cost emerges

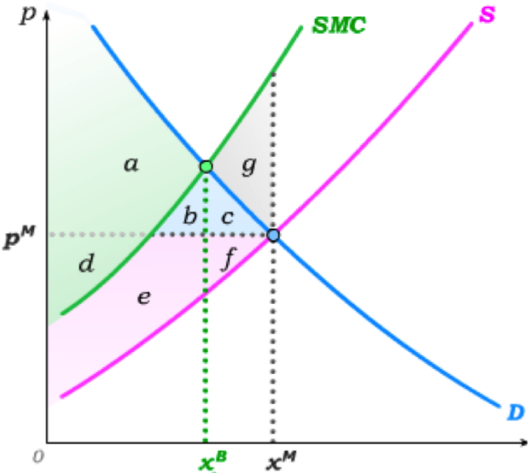
- Point: different (simple) models have different strengths and weaknesses

Illustration, equilibrium/soc. optimum



- Market equilibrium: intersection between supply and demand
- Social optimum: intersection between demand (marginal benefit) and social marginal cost
- In this case, too many goods are produced and purchased in the market equilibrium
- (Note that Nechyba's graphs are not linear...)

Illustration , dead weight loss



- Market equilibrium:
  - Consumer surplus: $\mathfrak{a}+\mathfrak{b}+\mathfrak{c}$
  - Producer surplus: $\mathfrak{d}+\mathfrak{e}+\mathfrak{f}$
  - Externality cost (CO^2): $-\mathfrak{b}-\mathfrak{c}-\mathfrak{e}-\mathfrak{f}-\mathfrak{g}$
  - *Net surplus:* $\mathfrak{a}+\mathfrak{d}-\mathfrak{g}$
- Social optimum :
  - *Net surplus:* $\mathfrak{a}+\mathfrak{d}$
- Dead weight loss from the externality: $\mathfrak{g}$

Mathematical analysis

- Market equilibrium given by: $x_d(p^M) = x_s(p^M)$, which yields:

$$p^M = \frac{\beta A - \alpha B}{\alpha + \beta} \qquad x^M = \frac{A+B}{\alpha + \beta}$$

- Let $p_d(x) = x_d^{-1}(x)$ be the inverse demand function, then social optimum is defined by the equation $SMC(x^{opt}) = p_d(x^{opt})$ which yields:

$$x^{opt} = \frac{A+B}{\alpha + \beta + 2\delta^2}$$

- We find (of course) that $x^M > x^{opt}$

Socrative quiz question

True or false: If demand is perfectly inelastic, a production externality creates no deadweight loss.

Summary

- Nechyba's externality model illustrates the same problem as the steel/fish example did:
  - Market equilibrium leads to too much pullution compared to the social optimum
  - This is because the firms do not take the externality cost into account
- Now: How can we restore efficiency with policy measures?

Production quotas

- A simple solution: Prohibit firms to produce more than x^{opt}
- Works on paper, but can be problematic in practise:
  - Policymaker will have to know x^{opt} (intersection between SMC and demand curve, which requires detailed knowledge of firms, technology etc.)
  - Another problem is to determine which firms should be permitted to produce

Pigouvian tax

- We saw that if you levy a tax, output will fall (slides 1a)
- Arthur Cecil Pigou's idea: Can a tax lower the equilibrium quantity to obtain the socially optimal level?
- Example for a case where the primary goal of a tax is not just to generate tax revenue

Pigou mathematics

- Levy a unit tax t; then market equilibrium becomes:

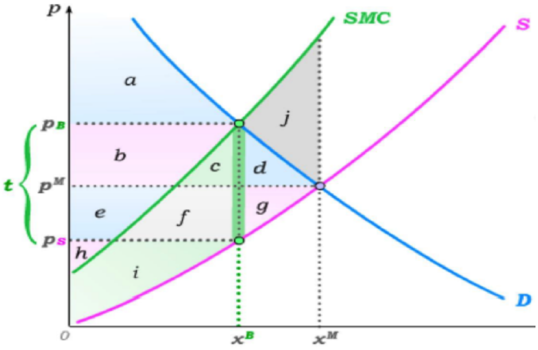
$$x^t = \frac{A+B-t}{\alpha+\beta} \qquad p_d^t = \frac{A\beta - B\alpha + \alpha t}{\alpha+\beta} \qquad p_s^t = \frac{A\beta - B\alpha - \beta t}{\alpha+\beta}$$

- We would like to set t such that we obtain the socially optimal quantity, that is $x^t = x^{opt}$; insert and solve for t to get:

$$t^{opt} = \frac{2\delta^2(A+B)}{\alpha+\beta+2\delta^2}$$

- Result: A Pigou tax set to t^{opt} will result in a market equilibrium that equals the social optimum

Pigou tax graphically



- Pigou tax moves the equilibrium down to the socially optimal level
- Market equilibrium with a Pigou tax:
    - Consumer surplus: $\mathfrak{a}$
    - Producer surplus: $\mathfrak{i} + \mathfrak{h}$
    - Externality cost (CO^2): $-\mathfrak{e} - \mathfrak{f} - \mathfrak{i}$
    - Tax revenue: $+\mathfrak{e} + \mathfrak{b} + \mathfrak{c} + \mathfrak{f}$
    - *Net surplus:* $\mathfrak{a} + \mathfrak{b} + \mathfrak{e} + \mathfrak{h}$
- Social optimum:
    - *Net surplus:* $\mathfrak{a} + \mathfrak{b} + \mathfrak{e} + \mathfrak{h}$

How does it work?

- Try to calculate the marginal CO^2 cost when the socially optimal level is produced:

$$MC_E(x^{opt}) = 2\delta^2 x^{opt} = \frac{2\delta^2(A+B)}{\alpha+\beta+2\delta^2}$$

- This is *exactly* the same as the optimal Pigou tax, t^{opt}!

- Intuition: Pigou tax works because it raises the marginal cost for firms, which internalizes the marginal external cost.

Pigou criticisms

- Reaching the social optimum requires that poliymakers know the marginal external costs at the socially optimal level
    - In principle, this means that you have to know the marginal external cost curve (and therefore also the optimal level of production)
- Pigou's idea was to tax production/sales; if we can measure the amount of emitted pollution per firm, we can also tax pollution directly
    - In Nechyba's model, the approaches are equivalent (tax of t per unit CO^2 = tax on δt per unit sold), but this does not always hold.

Socrative quiz question

Imagine the government only knows the marginal social costs of emitting CO^2 at the market equilibrium level and wants to set the tax equal to this level. When will this still yield the socially optimal outcome?

- a) When marginal social cost of emitting CO^2 increase linearly in the quantity produced.
- b) When marginal social cost of emitting CO^2 are constant.
- c) When the social cost of emitting CO^2 increase in the quantity produced.
- d) When the marginal social cost of emitting CO^2 are negative.

Steel and fish revisited I

- Go back to the steel/fish example and levy a tax t per unit of pollution, x:

$$\max_{s,x} = p_s \cdot s - c_s(s,x) - t \cdot x$$

- First order condition for interior solution (determined by s^* and x^*):

$$p_s = \frac{\partial}{\partial s} c_s(s^*, x^*) \quad , \quad \frac{\partial}{\partial x} c_s(s^*, x^*) = -t$$

- Remember that in social optimum it must hold that:

$$\frac{\partial}{\partial x} c_s(s', x') = -\frac{\partial}{\partial x} c_f(s', x')$$

- If we set $t = \frac{\partial}{\partial x} c_f(s', x')$ these become equivalent to the social optimum, so directly taxing pollution works

Steel and fish revisited II

- Instead levy a tax τ per unit of steel produced, s:

$$\max_{s,x} = p_s \cdot s - c_s(s,x) - \tau \cdot s$$

- First order condition for interior solution let s^* and x^* determine the solution):

$$p_s - \tau = \frac{\partial}{\partial s} c_s(s^*, x^*) \quad , \quad \frac{\partial}{\partial x} c_s(s^*, x^*) = 0$$

- We can generally *not* choose τ such that these equations become equivalent to the social optimum

Why does the tax on steel not work?

- The difference compared to Nechyba is that the steel firm can lower pollution without lowering production:
  - In Nechyba's model, firms will only have to determine the production level and then the amount of pollution will follow
  - By contrast, here the steel firm is choosing both how much to pollute and how much to produce
- Directly taxing pollution in the fish/steel economy: Incentive to reduce pollution (indirect effect on production)
- Pigou tax on steel in the fish/steel economy: Incentive to reduce steel production (indirect effect on pollution)

Nechyba model: Pigou example with numbers I (Do the calculations yourselves)

- Assume that $B = 0$, $\beta = 1$, $A = 100$, $\alpha = 1$, $\delta = 1$ and that $C_E(x) = x^2$
- Market equilibrium without tax: $p^M = 50$, $x^M = 50$
- Social optimum: $x^{opt} = 25$
- Marginal CO^2 cost in optimum (that is, optimal tax): $MC_E(25) = 50 = t^{opt}$

Pigou example with numbers II

- Market equilibrium with a Pigou tax of $t^{opt} = 50$: $p_d^t = 75$, $p_s^t = 25$, $x^t = 25$
- Welfare without tax: $CS = 1250$, $PS = 1250$, $C_E = 2500$, net surplus is 0
- Welfare with tax: $CS = 312$, $PS = 312$, $C_E = 625$, tax revenue is 1250; total net surplus is 1250

Tradable quotas (Cap and trade)

- If CO^2 emission can be measured at the firm-level, another policy solution is to establish a market for tradable emission quotas (permits):
  - The government issues a number of CO^2 quotas V
  - The government demands that every time a firm emits a unit of CO^2 it will have to possess the corresponding amount of quotas
  - The firms will therefore have to buy quotas corresponding to how much they emit
  - (We assume that the firms buy quotas directly from the government but there can also be trades between firms)

Nechyba's model with quotas

- Let r be the price for a quota; for every unit of steel the firm produces it will have to buy quotas corresponding to δr, giving the new marginal cost:

$$MC_k(x,r) = MC_s(x) + \delta r = -B + \beta x + \delta r$$

- The firm will produce up to the point where marginal cost equals price, so for a given price of quotas r, $MC_k(x,r)$ will be the new inverse supply curve

- For a given r we can find the equilibrium quantity traded on the market setting the inverse supply curve equal to the inverse demand curve:

$$MC_k(x^*, r) = x_d^{-1}(x^*) \quad \Rightarrow \quad x^*(r) = \frac{A + B - \delta r}{\alpha + \beta}$$

Demand for quotas

- We see (not too surprising) that quotas with a price r will result in the same equilibrium quantity as a Pigou tax of δr

- For every unit produced, a firm will have to buy δ quotas, so given that x^* is the socially optimal quantity, we can find the demand for quotas at the socially optimal level of production (as a function of the price) r):

$$D_v(r) = \delta x^*(r) = \delta \frac{A + B - \delta r}{\alpha + \beta}$$

- We can also find the inverse demand curve for emission quotas (as a function of the quantity of quotas v):

$$r(v) = \frac{\delta(A+B) - (\alpha+\beta)v}{\delta^2}$$

Equilibrium on the market for quotas

- The supply is easy to find: the government issues exactly V quotas so the supply is perfectly inelastic

- Equilibrium quantity is therefore V

- We can hence find the equilibrium price by inserting V in the inverse demand curve:

$$r^* = r(V) = \frac{\delta(A+B) - (\alpha+\beta)V}{\delta^2}$$

Social optimum

- Assume that $V < \frac{A+B}{\alpha+\beta}$ (explanation will follow!)
- In equilibrium, the firms will purchase V quotas; that is, they emit V units of CO^2 and produce (and sell) $x^* = \frac{1}{\delta}V$ goods
- Equilibrium price of quotas becomes: $r^* = \frac{\delta(A+B)-(\alpha+\beta)V}{\delta^2}$ (we can also derive the price of the good from here)
- Central observation: The government can precisely control equilibrium output and equilibrium CO^2; $V = \delta x^{opt}$ implements the social optimum

The price for quotas in social optimum

- Look at the price of quotas in equilibrium if we implement the social optimum, $V = \delta x^{opt} = \delta \frac{A+B}{\alpha+\beta+2\delta}$:

$$r^* = \frac{2\delta(A+B)}{\alpha+\beta+2\delta^2}$$

- This expression is exactly equal to the optimal Pigou tax, t^{opt} times $\frac{1}{\delta}$
  - The Pigou tax on the final good implements the socially optimal production level by levying a tax per unit of the final good that is equal to the marginal external cost.
  - A cap and trade system implements the socially optimal production level because the equilibrium price per quota becomes equal to the marginal external cost (divided by the emission per unit δ).

Socrative quiz question

True or false? Imagine there is a cap-and-trade system in place. The government could further reduce the equilibrium level of CO^2 emission if it introduces a tax on emission permits.

If many quotas are issued

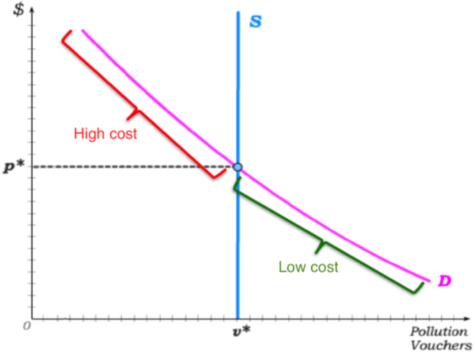
- All we did above was under the assumption that there were not too many quotas $V < \frac{A+B}{\alpha+\beta}$, but what if $V > \frac{A+B}{\alpha+\beta}$?
  - Note that this results in $V \geq \delta x^M$; there are enough quotas issued for the original market equilibrium to exist.
  - In this case, the math yields $r^* \leq 0$ and a negative price does not make much sense (remember that we could have written $r(V) = \max(\frac{\delta(A+B)-(\alpha+\beta)V}{\delta^2}, 0)$ to solve this problem)
  - Interpretation: If the government issues more permits than is needed for the original market equilibrium to exist, the equilibrium does not change at all...
  - ... and because permits are not a scarce resource their price will become 0

Permit critiques

- Social optimum requires that policymakers know the socially optimal level of CO^2 emission.
  - Note the difference to the Pigou tax where you had to know the marginal external cost
- Different options on how to distribute permits (auction, give them away for free, traded on a free market etc.) can be used politically because they change the welfare of consumers/firms

Permit trade and heterogeneous firms

- We could introduce a more complex model, where there are different types of firms with different costs of lowering the pollution ("high cost"vs "low cost")



- "High cost"⟶ always demand permits
- "Low cost"⟶ demand permits if the price is low
- The permits will be purchased by “high cost” firms ⟶ pollution is lowered in the cheapest possible way

Permit example with numbers

- Assume in Nechyba's model that $B = 0$, $\beta = 1$, $A = 100$, $\alpha = 1$, $\delta = 1$ and $C_E(x) = x^2$
- Market equlibrium without permits: $p^M = 50$, $x^M = 50$
- Social optimum: $x^{opt} = 25$
- The government issues the socially optimal amount of permits $V = \delta x^{opt} = 25$

Permit example with numbers II

- Equilibrium price and quantity on the market for permits: $V = 25$, $r^* = 50$ (the price for permits is equal to the Pigou tax from the previous example because $\delta = 1$)

- Equilibrium price and quantity on the market for goods: $x^* = 25$, $p^* = 75$

- Welfare without permits: $CS = 1250$, $PS = 1250$, $C_E = 2500$, net surplus is 0

- Welfare with permits: $CS = 312$, $PS = 312$, $C_E = 625$, revenue from selling permits is 1250; net surplus is 1250

Positive externalities and externalities arising from consumers

- We have focused on *negative production* externalities (*pollution*, noise, etc.)
- There also exist *positive* production externalities: more utility to consumers that firms do not take into account
  - Typical example is R&D: The more I invest in research and development, the more other firms will learn how to produce
  - You can analyze these example in the same way; Pigou taxes will become Pigou subsidies
- Finally there also exist positive and negative externalities arising from consumer behavior that can be analysed in the same way (e.g. fireworks).

Socrative Quiz Question

Which one of these gives firms a greater incentive to invest in research and innovation to make their production technology less emission-intensive?

- a) Pigouvian tax on the produced good.
- b) Tradable CO^2 permits.
- c) They give the same incentive.

What have we learned?

- What is an externality?
- Calculate the market equilibrium with externalities
- The effect of externalities on the efficiency of the market equilibrium
- How and when Pigou taxes can solve the problem of externalities
- How and when tradable permits can solve the problem of externalities
- Calculate the market equilibrium with Pigou taxes or tradable permits