Mikro II, lecture 1a

Distortionary taxes

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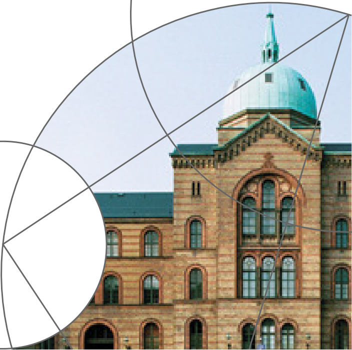


Plan for the lecture

1. Compute partial equilibria with perfect competition, with and without taxes
2. The effect of taxes on buyers and sellers (who pays the tax)
3. Compute tax revenue and welfare effects (dead weight loss)

Partial equilibrium with perfect competition

- All buyers and sellers take the price as given

- The demand curve indicates the demand as a function of price, $D(p)$
  - Derived from the consumer's utility maximization problem (holding income and other prices fixed): $D(p_1) = x^*(p_1, \overline{p_2}, \overline{I})$

- Supply curve indicates the supply as a function of price
  - Derived from firm's profit maximization problem (holding factor prices fixed): $S(p) = x^*(p, \overline{w})$

- (In some markets the sellers are consumers/households and the buyers are firms, for example in the labor market)

Socrative Quiz Question

For which type of good is demand increasing in its own price?

- a) Inferior good
- b) Giffen good
- c) Luxury good
- d) Public good
- e) Normal good

Equilibrium and the inverse functions

- If a price p^* satisfies that supply equals demand, we have an equilibrium (with a corresponding equilibrium output q^*)

$$D(p^*) = S(p^*) \quad (= q^*)$$

In this course (and most economics) we make equilibrium analysis: "How does the stable situation look like?"

- It is often useful to use the *inverse* supply and demand function:

$$p_d(q) = D^{-1}(q) \quad p_s(q) = S^{-1}(q)$$

Two extreme situations for supply

- Perfectly inelastic supply; vertical curve
  - A given quantity is supplied no matter what the price is (land, real estate?)
  - Supply function: $S(p) = k$; Inverse supply function is not defined
- Perfectly elastic supply; completely flat curve
  - At a given price, suppliers will supply an infinite amount of goods (imports?)
  - Inverse supply function: $p_s(q) = c$; Supply function not defined

Two extreme situations for demand

- Perfectly inelastic demand; vertical curve
  - A given quantity is demanded no matter the price
  - Demand function: $D(p) = k$; Inverse demand function not defined
- Perfectly elastic demand; completely flat curve
  - At a given price, buyers will buy an infinite amount, at higher prices they will not buy anything
  - Inverse demand function: $p_d(q) = c$; Demand function not defined

Equilibrium in the extreme scenarios

- If one curve is perfectly elastic, it determines the equilibrium price and the other curve determines the quantity, for example:

$$p_s(q) = c \Rightarrow p^* = c, \quad q^* = D(c)$$

- If one curve is perfectly inelastic, it determines the equilibrium quantity whereas the other curve will determine the price, for example:

$$S(p) = k \Rightarrow q^* = k, \quad p^* = p_d(k)$$

- (If both curves are perfectly elastic or perfectly inelastic, extreme situations will occur)

Socrative Quiz Question

True or false: An increase in households' income will have a stronger effect on the equilibrium price of a good the more elastic the supply curve.

Linear curves

- It is often useful to study situations where both demand and supply are linear:

$$D(p) = a - b \cdot p$$
$$S(p) = c + d \cdot p$$

- Certain assumptions regarding agent preferences and firm technology are indirectly embedded in these linear demand and supply functions.

- Sometimes these curves are used as local approximation for curves that are not completely linear (Taylor approximation).

- Note: Mathematically, the above function makes negative demand and supply possible; solution: $D(p) = \max(a - b \cdot p, 0)$

Equilibrium with linear curves

- Solve the above $D(p^*) = S(p^*)$ to find the equilibirum price:

$$p^* = \frac{a-c}{b+d}$$

- Insert this price in one of the above curves to obtain the equilibirum quantity $(D(p^*) = S(p^*) = q^*)$:

$$q^* = \frac{ad+bc}{b+d}$$

- Under the assumption that $a > c$ and $b, d > 0$, both p^* and q^* are non-negative (no issues with negative prices and/or quantities)

Comparative statics

- Classic analysis in economics: how does the equilibrium change if the exogenous variables a, b, c, d change?

$$q^* = \frac{ad + bc}{b + d} \qquad p^* = \frac{a - c}{b + d}$$

- Often this is very easy to see right away. If not you can find the derivative (or illustrate the scenario graphically)

- For example: $b \uparrow$ (that is: greater slope of the demand curve $\Rightarrow$ $p^* \downarrow$ (that is: lower equilibrium price); and for an unchanged supply curve, lower quantity $q^* \downarrow$

- Similarly: $d \uparrow \Rightarrow p^* \downarrow$ and $q^* \uparrow$

Distorting taxes

- We will now analyse what will happen if we introduce taxes
- Interesting questions:
  - How does a tax influence equilibrium price and quantity?
  - How much tax revenue is collected?
  - Which implications does a tax have for social welfare?

Taxes: The math

- A tax creates a wedge between the price that the buyer pays and the price that the seller collects; important point: *There will now be two different prices*

- Producer price, p_s: what the seller collects after paying the tax

- Consumer price, p_d: what the buys pays after paying the tax

- A tax t on every good sold: $p_d = p_s + t$

  A tax t that is proportional to the price: $p_d = p_s(1+t)$

Reminder: *Statutory incidence*

- The government specifies who legally has to pay the tax ("statutory incidence")
- A known result from the course Principles of Economics: It does not matter who legally has to pay the tax for who bears the cost ("economic incidence")
- Can be proven graphically and mathematically

Mathematics: *Statutory incidence*

- Demand depends on what the consumer has to pay ($D(p_d)$)
- Supply depends on what the producer receives ($S(p_s)$)
- If the consumer pays, we have: $p_d = p_s + t$ or in equilibrium: $p_d(q_t^*) = p_s(q_t^*) + t$
- If the producer pays, we have: $p_d - t = p_s$ or in equilibrium: $p_d(q_t^*) - t = p_s(q_t^*)$
- Note that the two equations are identical (t can be moved to the other side of the equation)

Taxes and linear curves

- Let us find the equilibrium in a situation where the demand and supply are linear and a unit tax t is introduced:

$$D(p_d) = a - b \cdot p_d \qquad S(p_s) = c + d \cdot p_s$$

$$p_d^* = p_s^* + t \qquad D(p_d^*) = S(p_s^*)$$

- Replacing p_d^* in the demand curve and using the equilibrium condition, we can solve for the equilibrium producer price: $p_s^* = \frac{a-c-bt}{b+d}$

- The consumer price becomes: $p_d^* = \frac{a-c+dt}{b+d}$

- Insert this in the demand or supply curve to obtain the equilibrium quantity: $q_t^* = \frac{ad+bc-dbt}{b+d}$

How are producers affected?

$$p_s^* = \frac{a - c - bt}{b + d}$$

- Comparative statistics: How does the producer price change in equilibrium due to the tax t? ($t = 0$ is the same as no tax)
- The tax pushes the producer price down: $\frac{\partial p_s^*}{\partial t} = -\frac{b}{b+d} < 0$
- The price is reduced by more the more elastic the demand curve: $\frac{\partial p_s^*}{\partial t} = -\frac{1}{1+\frac{d}{b}}$ is decreasing in b

How are consumers affected?

$$p_d^* = \frac{a - b + dt}{b + d}$$

- Comparative statistics: how does the consumer price change due to the tax t?
- The tax pushes the consumer price up: $\frac{\partial p_d^*}{\partial t} = \frac{d}{b+d} > 0$
- The consumer price increases more strongly if the supply curve is more elastic: $\frac{\partial p_d^*}{\partial t} = \frac{1}{\frac{b}{d}+1}$ is increasing in d

How is the traded quantity affected?

$$q_t^* = \frac{ad + bc - dbt}{b + d}$$

- Comparative statistics: how does the equilibrium quantity change due to the tax?
- The tax lowers the quantity: $\frac{\partial p_d^*}{\partial t} = -\frac{db}{b+d} < 0$
- The more price elastic demand and/or supply, the greater the reduction in quantity

Socrative Quiz Question

True or false: An additive unit tax can lead a market to break down.

Taxes with perfectly (in)elastic curves?

- If one side of the market is perfectly elastic, the whole tax will be paid by the other side of the market:

$$p_s(q) = c \Rightarrow p_s^* = c, \quad p_d^* = c + t$$

- If one side of the market is perfectly inelastic, the whole tax will be paid by that side of the market:

$$S(p_S) = k \Rightarrow p_d^* = p_d(k) \quad p_s^* = p_d^*(k) - t$$

Economic incidence

- Economic incidence tells us who really pays the cost of a tax (how producer and consumer prices are influenced by a tax).
- Previous slides have shown that the economic incidence depends on price elasticities
- “Rule of thumb”: The **less elastic demand**, and the **more elastic supply**, the greater the fraction of the tax that is paid by the **consumers**.

Economic incidence (the math)

- This rule can be proven mathematically
- Nechyba's proof is based on differentials (treating dt and dx as variables)
- This can also be done simply by differentiating the equilibrium condition (see next slide)

Derivation I

- Equilibrium prices with a tax t are p_s^*, p_d^* (depends on t)
- Equilibrium price without tax ($t = 0$) is p^*
- Equilibrium condition (in quantities), insert and differentiate wrt t:

$$
\begin{aligned}
D(p_d^*) &= S(p_s^*) && \Longleftrightarrow \\
D(p_d^*) &= S(p_d^* - t) && \Rightarrow \\
D'(p_d^*)\frac{\partial p_d^*}{\partial t} &= S'(p_d^* - t)\frac{\partial p_d^*}{\partial t} - S'(p_d^* - t) && \Longleftrightarrow \\
\left(D'(p_d^*) - S'(p_d^* - t)\right)\frac{\partial p_d^*}{\partial t} &= -S'(p_d^* - t) && \Longleftrightarrow \\
\frac{\partial p_d^*}{\partial t} &= -\frac{S'(p_d^* - t)}{D'(p_d^*) - S'(p_d^* - t)}
\end{aligned}
$$

Derivation II

$$\frac{\partial p_d^*}{\partial t} = -\frac{S'(p_d^* - t)}{D'(p_d^*) - S'(p_d^* - t)} \iff \frac{\partial p_d^*}{\partial t} = -\frac{S'(p_d^* - t)\frac{p^*}{q^*}}{D'(p_d^*)\frac{p^*}{q^*} - S'(p_d^* - t)\frac{p^*}{q^*}} \Rightarrow$$

$$\left.\frac{\partial p_d^*}{\partial t}\right|_{t=0} = -\frac{S'(p^*)\frac{p^*}{q^*}}{D'(p^*)\frac{p^*}{q^*} - S'(p^*)\frac{p^*}{q^*}} \iff \left.\frac{\partial p_d^*}{\partial t}\right|_{t=0} = -\frac{S'(p^*)\frac{p^*}{S(p^*)}}{D'(p^*)\frac{p^*}{D(p^*)} - S'(p^*)\frac{p^*}{S(p^*)}} \iff$$

$$\left.\frac{\partial p_d^*}{\partial t}\right|_{t=0} = -\frac{\varepsilon_s}{\varepsilon_d - \varepsilon_s}$$

Economic incidence result

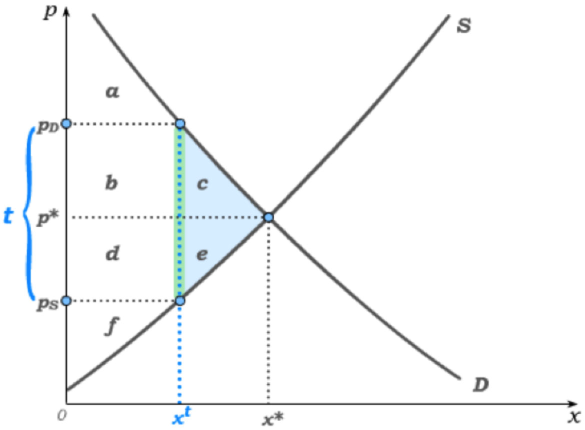
$$\left.\frac{\partial p_d^*}{\partial t}\right|_{t=0} = -\frac{\varepsilon_s}{\varepsilon_d - \varepsilon_s}$$

- Perfectly inelastic demand ($\varepsilon_d = 0$): consumers pay the tax ($\frac{\partial p_d^*}{\partial t} = 1$)
- Perfectly inelastic supply ($\varepsilon_s = 0$): firms pay the tax ($\frac{\partial p_d^*}{\partial t} = 0$)
- Demand and supply are equally elastic ($\varepsilon_s = -\varepsilon_d$): they share the tax ($\frac{\partial p_d^*}{\partial t} = \frac{1}{2}$)
- We have ignored the functional form of supply and demand: this result holds for every supply and demand curve (for small taxes)
- **Intuition: The most elastic side of the market can change behaviour most easily to avoid the tax.**

Welfare and tax revenue

- We have now seen how we can calculate market equilibrium after implementing a tax (partial equilibrium)
- We have also seen how the economic incidence is heavily connected to **price elasticities**.
- Next steps:
  - How do we calculate the tax revenue?
  - How do we calculate welfare changes?

Surplus squares and triangles



(Figure from Nechyba: slide notation replaces x with q)

- Without tax:
  - Consumer surplus: $\mathfrak{a}+\mathfrak{b}+\mathfrak{c}$
  - Producer surplus: $\mathfrak{e}+\mathfrak{d}+\mathfrak{f}$
- With tax
  - Revenue: $\mathfrak{b}+\mathfrak{d}$
  - Consumer surplus: $\mathfrak{a}$
  - Producer surplus: $\mathfrak{f}$
  - "dead weight loss": $\mathfrak{c}+\mathfrak{e}$
- Linear curves $\Rightarrow$ all fields are squares or triangles $\Rightarrow$ easy to compute

Socrative Quiz Question

True or false: If demand is perfectly inelastic, there will be no deadweight loss from a unit tax, but if supply is perfectly inelastic there will still be a deadweight loss.

Dead weight loss I

- A triangle with base t and height $q^* - q_t^*$, area formula: $\frac{1}{2} \cdot t \cdot (q^* - q_t^*)$
- Insert the previous expression for q^* and q_t^* for linear curves (with *parametres* a, b, c, d):

$$DWL(t) = \frac{db}{2(b+d)} t^2$$

- Point 1: Dead weight loss increases as the square of the tax (very small taxes result in a very *very* small loss in welfare)
- Attention: All conclusions regarding dead weight loss presented here are only valid if demand and supply curves are linear. Otherwise they serve as (good) linear approximations for all types of demand and supply (and small taxes)

Dead weight loss II

- Point 2: Dead weight loss is increasing in b and d $(DWL(t) = \frac{d}{2(1+\frac{d}{b})}t^2 = \frac{b}{2(\frac{b}{d}+1)}t^2)$

- b and d determine the slope (elasticity) of supply and demand curves: more price elastic consumers and firms $\Rightarrow$ greater dead weight loss

- Point 3: In the extreme scenario where either supply or demand is perfectly inelastic (b or d equal to 0) there is no dead weight loss

- Intuition: The dead weight loss occurs because some gains of trade cease to exist and a high elasticity results in a more severe drop in traded quantity

Tax revenue I

- A rectangle with the sides t and q_t^*, the area formula: $t \cdot q_t^*$

- Insert the above expression for q_t^* derived from the linear curve example to get:

$$TR(t) = \frac{ad+bc}{b+d}t - \frac{bd}{b+d}t^2$$

- Graphically this will be a quadratic function with the branches turning downwards. It is maximised where $t = \frac{ad+bc}{2bd}$

- For $t > \frac{ad+bc}{2bd}$ we have that a higher tax will lead to a fall in tax revenue; We have derived the famous Laffer-curve!

Tax revenue II

- We have seen that more elastic demand or supply $\Longrightarrow$ Lower tax revenue
- The intuition is very much the same as we saw before with the dead weight loss: High price elasticity $\Longrightarrow$ Large fall in the quantity $\Longrightarrow$ Less goods to tax!

The area and non-linear curves

- What if we would like to calculate consumer surplus, producer surplus or dead weight loss if demand and supply curves are non-linear?

- Use integrals! Integrals are the area underneath (between) functions, for example:

$$DWL(t) = \int_{q_t^*}^{q^*} (p_d(q) - p_s(q))\, dq$$

- Conceptually very simple, but it can take time

Dead weight loss revisited

- Up until now we have computed dead weight loss as we did in Principles of Economics, that is, by comparing consumer and producer surplus and tax revenue before/after the tax
  - The change in **producer surplus** is the same as the change in profit for the firm (supply curve is equal to marginal costs MC)
  - The (change in) **tax revenue** is the extra money that the government receives
  - The change in **consumer surplus** is change in consumer welfare due to the tax

- BUT: In Microeconomics you saw that this is actually only true for quasi linear preferences!

Microeconomics I recap

- When calculating dead weight loss we want to ask ourselves: How much money (can) we take from the consumer to make him just as well off as after the tax?

  (Alternatively: How much money is the consumers willing to pay to get rid of the tax?)

- This is what we call equivalent variation (EV)

- With quasi linear preferences the change in consumer surplus = EV so if the consumers we study have quasi linear preferences we've been spot on

- With other preferences this is NOT the case; the difference is due to income effects (quasi linear preferences $\Rightarrow$ no income effects)

Nechyba's example

- Nechyba focuses on an important problem with using consumers surplus as a measurement for welfare changes: taxing the labor market
  - The consumer will have to choose how much to consume and how much to work (how much leisure she wants)
  - The consumer takes the wage w as given (price of leisure), regular consumption will have the price 1 by definition
  - There is one firm that demands manpower (working hours) which is given by the demand curve $D(w)$ (comes from the firm's profit maximisation problem)

Dead weight loss using consumer surplus

- When measuring the price (wage) elasticity of labor supply in reality, researchers often find a very small number

- Let us hence assume that the supply of labor is perfectly inelastic

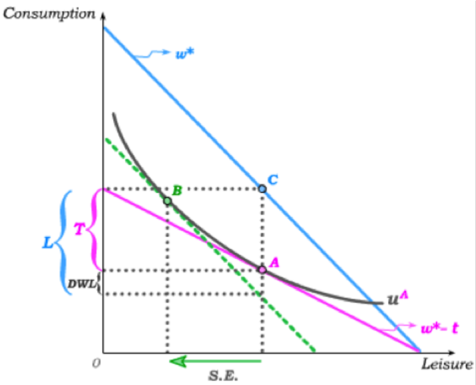
- Let us now try to calculate the dead weight loss from introducing a tax t per working hour using the uncompensated labor supply curve.

- This is easy! From previous slide: Perfectly inelastic supply $\Longrightarrow$ the supply side pays the whole tax and there will not be any dead-weight loss.

Recap: Labor supply and the Slutsky equation

- The Slutsky equation: The change in labor supply when the wage (after tax) decreases is a net effect that is covered by the sum of:
  - Negative substitution effect: you get paid less for each hour and will therefore work less
  - Positive income effect: for a given labor supply you have a lower income which leads you to work more (consume less leisure, which is a normal good)
- Calculating dead weight loss using the uncompensated supply curve is only precise if preferences are quasi-linear, that is, if the income effect is equal to zero
- A more realistic scenario: labor supply is inelastic because large substitution and income effects cancel each other out

Calculating dead weight loss using equivalent variation



- EV-analysis:
  - C: labor supply before tax
  - A: labor supply after tax
  - B: same utility after tax but with the old price system (before tax)
  - EV: How far does the old budget line have to be pushed down to reach B?

- The consumer is willing to pay L to get rid of the tax, the tax yields T in tax revenue, but since $L > T$ there will be a dead weight loss

Nechyba's points

- Calculations based on the uncompensated demand curve are only precise if the consumer has quasi-linear preferences (no income effects).

- Many situations with (almost) perfectly inelastic demand/supply are due to substitution and income effects cancelling each other out.

- So you have to be careful by concluding that there is no (or a very small) dead weight loss just because the (uncompensated) price elasticity is zero (or small).

- (A precise calculation of dead weight loss based on areas can be done by using the *Hicks compensated* demand instead, see Nechyba figure 19.6 c)

Socrative Quiz Question

If preferences are **not** quasi-linear, and one calculates the deadweight loss from a unit tax t on a good from the uncompensated demand curve, one will usually . . .

- a) . . . underestimate the deadweight loss regardless of whether the good is an inferior or a normal good.
- b) . . . overestimate the deadweight loss regardless of whether the good is an inferior or a normal good.
- c) . . . overestimate the deadweight loss if the good is an inferior good.
- d) . . . overestimate the deadweight loss if the good is a normal good.

What have we learned?

- Calculate partial equilibrium with and without taxes
- Calculate consumer surplus, producer surplus, tax revenue and dead weight loss
- Statutory incidence, economic incidence and their relation to elasticities
- Tax revenue and dead weight loss and their relation to elasticities
- The issues with calculating welfare changes using the uncompensated demand curve when preferences are not quasi-linear