UNIVERSITY OF COPENHAGEN

Mikro II, lecture 12a

Social utility

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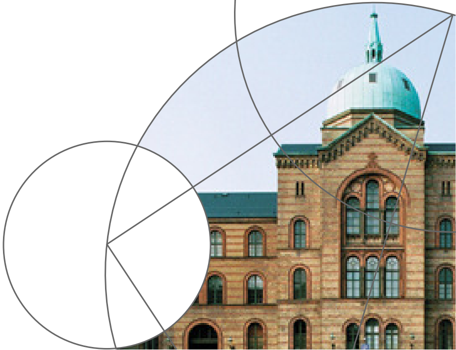


Plan for the lecture

1. Social utility and social welfare functions
2. Social utility maximization and its relation to efficiency and equality
3. John Rawl's thoughts on social utility

Agenda

- We investigate choices made at the social level (e.g. on taxation)
- Last slideshow: How can / should we aggregate individual preferences into a social preference?
- Now more purely normative approach: how should we assess whether a given situation in society is ”good”
- Obvious link to philosophy and ethics

Setup

- We will again use our standard Edgeworth economy
- 2 agents, 2 items, a given total amount of each good
- What is the right way to distribute the goods?
- How should we compare different distributions?

Example: Edgeworth economy I

Technology and Preferences	Behavior and Equilibrium
Exogenous func./var./relationships: $u_A(x_1^A, x_2^A)$ $u_B(x_1^B, x_2^B)$ $\alpha, \beta, e_1^A, e_2^A, e_1^B, e_2^B$ $e_1^A + e_1^B \geq x_1^A + x_1^B$ $e_2^A + e_2^B \geq x_2^A + x_2^B$ *Endogenous variables:* (x_1^A, x_2^A), (x_1^B, x_2^B)	*The decisions of the agents:* $\hookrightarrow$ *Conditional behavior:* *Equilibrium Conditions:*

Edgeworth, extra assumptions

- Different to previously, we explicitly allow that some of the goods end up not being consumed (free disposal)
- In addition, we assume that both utility functions are continuous and increasing
- Finally, we want to normalize the utility functions such that no consumption yields zero utility:

$$u_A(0,0) = u_B(0,0) = 0$$

Utilitarianism (utility ethics, utility philosophy)

- Utilitarianism is most often attributed to Jeremy Bentham (1748-1832): In order to assess right and wrong, we must look at whether we make (many) people happy:

  "It is the greatest happiness of the greatest number that is the measure of right and wrong."

- Another quote, from the other great utilitarian John Stuart Mills:

  "A sacrifice which does not increase, or tend to increase, the sum total of happiness, is considered as wasted."

- Utilitarianism takes its name from another word used for "happiness": *utility*

We need to evaluate people's utility

- Translating these ideas into our economic models: We must distinguish right and wrong from people's utility

- First step: What combinations of benefits are possible for us to achieve?

- Utility Possibility Set (UPS):

$$UPS = \left\{ u'_A, u'_B \,|\, u'_A = u_A(x_1^A, x_2^A) \text{ and } u'_B = u(x_1^B, x_2^B) \text{ and } (x_1^A, x_2^A, x_1^B, x_2^B) \in X \right\}$$

where X is the amount of possible states (consumption possibility frontier)

$$X = \left\{ (x_1^A, x_2^A, x_1^B, x_2^B) \geq 0 \,|\, e_1^A + e_1^B \geq x_1^A + x_1^B \text{ and } e_2^A + e_2^B \geq x_2^A + x_2^B \right\}$$

Utility Possibility Frontier I

- To find UPS, it is useful to first consider the Utility Possibility Frontier: The combinations of utility where A is made as well off as possible given B's utility

- Formally, we look at a maximization problem that we have looked at before:

$$u_A^*(u_B^*) = \max_{(x_1^A, x_2^A, x_1^B, x_2^B) \in A} u_A(x_1^A, x_2^A)$$

$$\text{s.t.}$$

$$u_B(x_1^B, x_2^B) = u_B^*$$

$$(x_1^A, x_2^A, x_1^B, x_2^B) \in X$$

Utility Possibility Set

- The solution to the problem from before can be seen as a function $u_A^*(u_B^*)$: If B is to have the utility u_B^*, what is the highest utility A can get?

  (The function will only be defined for utility levels u_B^* that B can actually achieve)

- We can map this function into a u_A, u_B diagram

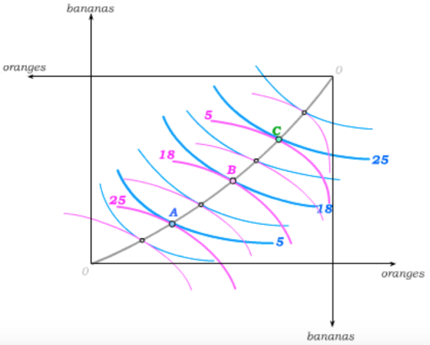
Contract curve and UPF I

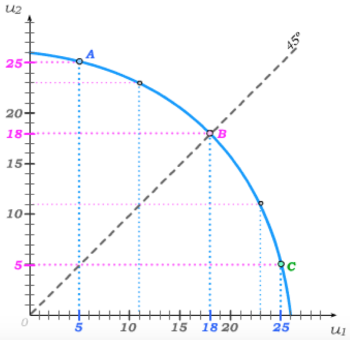
- Last time we looked at the maximization problem that defines $u_A^*(u_B^*)$ we concluded:

$$(x_1^A, x_2^A, x_1^B, x_2^B) \text{ is a solution } \iff (x_1^A, x_2^A, x_1^B, x_2^B) \text{ is (Pareto) efficient}$$

- The UPF is thus the amount of efficient states; we used to call it the *contract curve*

Contract curve and UPF II





- The contract curve shows the efficient states in the Edgeworth box. UPF shows the efficient states in the utility coordinate system

Socrative Quiz Question

True or false: If the two goods are perfect substitutes for the two agents, then the utility possibility frontier will be linear (a downward sloping line).

Social utility maximization I

- UPF indicates which combinations of utility we can choose from
- How should we choose?
- The utilitarianists talked about *"the sum total of happiness"*
  Aha! We must maximize the overall benefit:

$$\max_{u'_A, u'_B} u'_A + u'_B \qquad \text{s.t.} \qquad (u'_A, u'_B) \in UPS$$

- Note: choose a combination of utility = select a state; equivalent maximization problem:

$$\max_{x_1^A, x_2^A, x_1^B, x_2^B} u_A(x_1^A, x_2^A) + u_B(x_1^B, x_2^B) \qquad \text{s.t.} \qquad (x_1^A, x_2^A, x_1^B, x_2^B) \in X$$

Social utility maximization II

$$\max_{u'_A, u'_B} u'_A + u'_B \qquad \text{s.t.} \qquad (u'_A, u'_B) \in UPS$$

- Note the similarity to the consumer problem:
  - While the consumer maximizes his own utility; here we maximize the sum of total utility
  - The consumer must choose something from the budget set; here we have to choose something from UPS
- We can do a graphical analysis: Define total utility - "indifference curve" as the number of points that keep the utility sum constant equal to U:

$$\{u'_A, u'_B | u'_A + u'_B = U\}$$

Social utility maximization III

- As usual, there are many ways to solve the same maximization problem

- A useful way is to select states instead of utility:

$$\max_{x_1^A, x_2^A, x_1^B, x_2^B} u_A(x_1^A, x_2^A) + u_B(x_1^B, x_2^B) \qquad \text{s.t.} \qquad (x_1^A, x_2^A, x_1^B, x_2^B) \in X$$

- The constraint here is that the state is possible; insert and get:

$$\max_{x_1^A, x_2^A} u_A(x_1^A, x_2^A) + u_B(e_1^A + e_1^B - x_1^A, e_2^A + e_2^B - x_2^A)$$

Social utility maximization IV

$$\max_{x_1^A, x_2^A} u_A(x_1^A, x_2^A) + u_B(e_1^A + e_1^B - x_1^A, e_2^A + e_2^B - x_2^A)$$

- The FOCs:

$$\frac{\partial u_A}{\partial x_1} - \frac{\partial u_B}{\partial x_1} = 0 \iff \frac{\partial u_A}{\partial x_1} = \frac{\partial u_B}{\partial x_1}$$
$$\frac{\partial u_A}{\partial x_2} - \frac{\partial u_B}{\partial x_2} = 0 \iff \frac{\partial u_A}{\partial x_2} = \frac{\partial u_B}{\partial x_2}$$

- Two equations with two unknowns (x_1^A, x_2^A) that can be solved, intuitive interpretation

Social Utility Function I

- Inspired by the utilitarian philosophers, we have developed a way to choose the socially “best” condition

- Define $U(u_A, u_B) = u_A + u_B$ and we can talk about $U(u_A, u_B)$ as a social welfare function (SWF)

- SWF indicates how good a given state is so we can compare states and / or select the optimal one (typically only one solution):

$$\max_{u'_A, u'_B} \quad U(u'_A, u'_B) \qquad \text{s.t.} \qquad (u'_A, u'_B) \in UPS$$

Social Utility Function II

- Our SWF above was a sum function (just like perfect substitutes), but we have seen many other examples of consumer utility; we can also find many possible SWFs:
  - Bentham / Pareto (1-to-1 perfect substitutes): $U(u_A, u_B) = u_A + u_B$
  - *Harsanyi* weights (other perfect substitutes): $U(u_A, u_B) = \gamma_A u_A + \gamma_B u_B$
  - Cobb-Douglas: $U(u_A, u_B) = u_A^{\beta} u_B^{1-\beta}$
  - Rawls (Leontief, more later): $U(u_A, u_B) = \min\{u_A, u_B\}$
  - Etc...

Equality and SWF

- The Bentham SWF maximizes the sum of benefits: if we can make a single agent extremely well off, this is better than making both agents a little well off (i.e. Bentham $\Rightarrow$ equality in utility is not important)

- Other SWFs tend to assume their greatest value when the utility is not too far from each other
  - If the SWF is more ”concave” ...
  - .. the indifference curves are more “curved” (remember consumer interpretation regarding “love of variety”)
  - We could also use a convex SWF: then we would prefer more inequality!

- Different SWFs can be interpreted as reflecting different attitudes towards inequality

Pareto efficiency and SWF I

- However, why have we only looked at (Pareto) efficiency so far and not used SWF?

Social utility maximization requires Pareto efficiency

Let $U(u_A, u_B)$ be a SWF that is strictly increasing in both arguments (higher utility is good). Let $(x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime})$ be a solution to the social utility maximization problem:

$$\max_{x_1^A, x_2^A, x_1^B, x_2^B} (u_A(x_1^A, x_2^A), u_B(x_1^B, x_2^B)) \qquad \text{s.t.} \qquad (x_1^A, x_2^A, x_1^B, x_2^B) \in X$$

Then $(x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime})$ is a Pareto efficient (Pareto optimal) state

Pareto efficiency and SWF II

- If we cannot agree on a precise SWF, then the Pareto Efficiency is a minimum condition for social utility maximization; proof sketch:
  - Assume that $(x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime})$ is not Pareto Optimal
  - Then there is another state $(x_1^{A\prime\prime}, x_2^{A\prime\prime}, x_1^{B\prime\prime}, x_2^{B\prime\prime})$ that makes one of the agents better without making the other worse off
  - But then $(x_1^{A\prime\prime}, x_2^{A\prime\prime}, x_1^{B\prime\prime}, x_2^{B\prime\prime})$ yields a strictly higher social utility (SWF is strictly increasing)
  - Bottom line: $(x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime})$ Is not a solution to the maximization problem

Second-best UPS

- It is worth noting that in our deduction of the UPS (utility area) we assumed that we (the society) could freely choose any condition
- This is a reasonable assumption if we have ”redistribution” or lump sum taxes
- In practice, proportional taxes are often used that distort and create inefficiency
- In that case, the relevant scope of use may be smaller: Second best UPS, see Nechyba

Socrative Quiz Question

True or False: If taxation is distortionary, social utility maximization will tend to favor more equal distributions.

Social Choice Functions revisited

- We can link these ideas to our discussion of social preferences from last lecture.

- Define a preference relation for agents A and B by:

$$(x_1^A, x_2^A, x_1^B, x_2^B) \succsim_A (x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime}) \iff u_A(x_1^A, x_2^A) \geq u_A(x_1^{A\prime}, x_2^{A\prime})$$
$$(x_1^A, x_2^A, x_1^B, x_2^B) \succsim_B (x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime}) \iff u_B(x_1^B, x_2^B) \geq u_A(x_1^{B\prime}, x_2^{B\prime})$$

- Let $U(u_A, u_B)$ be a SWF (e.g. Bentham) and define a social preference by:

$$(x_1^A, x_2^A, x_1^B, x_2^B) \succsim^* (x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime}) \iff$$
$$U(u_A(x_1^A, x_2^A), u_B(x_1^B, x_2^B)) \geq U(u_A(x_1^{A\prime}, x_2^{A\prime}), u_B(x_1^{B\prime}, x_2^{B\prime}))$$

Arrow was wrong?

- Now we have aggregated the individual preferences; are Arrow's axioms complied with?

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Arrow was wrong?

- Now we have aggregated the individual preferences; are Arrow's axioms complied with?
- Have we then found a contradiction with Arrow's impossibility theorem?
- The process here starts from the agents' utility functions, not their preference relations
- Remember that a social choice function starts from preference relations

The problem with SWF I

- This is *not* just mathematical ingenuity: assume a new utility for A: $v_A(x_1^A, x_2^A) = 2 \cdot u_A(x_1^A, x_2^A)$

- This does not change the preferences of A:

$$(x_1^A, x_2^A, x_1^B, x_2^B) \succsim_A (x_1^{A\prime}, x_2^{A\prime}, x_1^{B\prime}, x_2^{B\prime}) \iff v_A(x_1^A, x_2^A) \geq v_A(x_1^{A\prime}, x_2^{A\prime}) \iff u_A(x_1^A, x_2^A) \geq u_A(x_1^{A\prime}, x_2^{A\prime})$$

- But this will change how our SWF assesses conditions and thus change our social preferences:

$$U(v_A(x_1^{A\prime}, x_2^{A\prime}), u_B(x_1^{B\prime}, x_2^{B\prime})) = U(\mathbf{2 \cdot u_A(x_1^{A\prime}, x_2^{A\prime})}, u_B(x_1^{B\prime}, x_2^{B\prime}))$$

The problem with SWF II

- So for someone with the *same preferences*, the exact utility we use makes a difference to our assessments of different states

- The problem here is that with SWF we throw ourselves into *cardinal* utility comparisons: we take the specific utility level seriously and compare across people

- So far (and very explicitly in Micro I) we have considered utility purely *ordinal*: only the relative utility between two options matters

Problem also discussed in philosophy

- This problem of utilitarianism (utility ethics, utility philosophy) has also been widely discussed among philosophers
- Main criticism / debate: What is utility? How should it be defined? How should it be measured?
- That's exactly the problem we just encountered in math

Cardinal utility and equality

- It is worth noting that the problem of cardinality and the choice of utility function also has an impact on our discussion of equality

- Note that we can use monotonous transformations to make the utility function more concave / convex without changing preferences, for example:
  $v_A(x_1^A, x_2^A) = \left(u_A(x_1^A, x_2^A)\right)^2$ or $v_A(x_1^A, x_2^A) = \left(u_A(x_1^A, x_2^A)\right)^{\frac{1}{2}}$
  - A more concave utility means faster declining marginal utility ...
  - ... and generally a "rounder" UPS
  - Which, overall, will result in social utility maximization involving more equality (and vice versa for convex utility)

John Rawls

- We end by linking our SWF discussion (with all its problems) to another important philosopher: John Rawls (1921-2002)

- Very simplified, Rawl's attitude on how to distinguish right and wrong can be summed up by assessing every situation based on whether it improves the situation of the worst-off
"those who benefit least have ... a veto"

- As seen earlier, we can formalize the idea in Rawls SWF that sets equality very high (and which, incidentally, is *not* strictly growing):

$$U(u_A, u_B) = \min\{u_A, u_B\}$$

Rawls' Veil of Ignorance I

- Rawls, incidentally, had an interesting idea of how to arrive at ”the right thing to do”
- Rawls thought we should (imagine to) meet in a situation where we know what society will look like, but not know who in society we each become:
  - You and I know that in a little while we will be ”born into” our little Edgeworth economy
  - We will be born either as Agent A or B, but we do not know which of them
  - Now we have to figure out how to organize the economy (how much redistribution, who needs what, etc.)

Rawls' Veil of Ignorance II

- Besides being an interesting thought experiment, the Veil of Ignorance is relevant because we can relate the idea to our ideas of utility maximzation.
- We organize our economy while we are uncertain whether we will become A and get the utility u_A or become B and get utility u_B
- Our decision under uncertainty would be such as to maximize expected utility:

$$\max \quad P(\text{born as } A) \cdot u_A + P(\text{born as } B) \cdot u_B$$

Rawls' Veil of Ignorance III

- But maximizing expected utility here is equivalent to not being risk averse with respect to one's level of utility.

- We can incorporate some risk aversion by assessing the utility of each condition based on a concave transformation, e.g. u:

$$\max \quad P(\text{born as } A) \cdot \sqrt{u_A} + P(\text{born as } B) \cdot \sqrt{u_B}$$

- A more extreme form of risk aversion is *minimax* behavior: maximizing the worst that can happen; if used here it actually gives us exactly Rawls' SWF:

$$\max \min\{u_A, u_B\}$$

Socrative Quiz Question

True or false? Maximization of the following would lead to a more unequal distribution, but would still guarantee positive utility levels for both A and B.

$$\max \quad P(\text{born as } A) \cdot u_A^2 + P(\text{born as } B) \cdot u_B^2$$

A little more philosophy

- We end by briefly going back to philosophy and ethics
- All of our approaches to right and wrong here are based solely on looking at the consequences of what society chooses
- An alternative approach is that we should care about the process by which something comes about, not the exact consequences.
- This approach would say that it can be okay sometimes to end up with something very unequal (or bad) if the process has been good (free? fair?)

What have we learned?

- What is a Social Welfare Function (SWF)
- Solve problems with SWFs and find the social utility maximum
- The relationship between social utility maximization, Pareto optimality and efficiency
- The relationship between SWFs and attitudes towards equality