UNIVERSITY OF COPENHAGEN

Mikro II, lecture 10b

Public Goods: Problems and Solutions

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Plan for the lecture

1. A bit more about public goods and inefficiency
2. Lindahl equilibrium with public goods
3. Mechanism design and the Vickrey-Clarke-Groves mechanism

Reminder

- Reminder: We have looked at an (Edgeworth) economy with 2 agents where one of the goods was public

- We saw that a Nash equilibrium in which the agents themselves produced the public good was inefficient by comparing the condition of efficiency with the condition of individual utility:

$$|MRS_A| + |MRS_B| = c$$

$$\text{vs.}$$

$$|MRS_A| = c$$

$$|MRS_B| = c$$

- We start by looking at a few ways to think about this inefficiency

Public goods as a problem of externality

- The equations show that the inefficiency arises because each agent only takes into account their own willingness to pay for the public good
- Very similar to our discussion of externalities: When an agent produces / buys more public goods there is a positive externality that the agent does not internalize
- Our insights from the subject of externalities can thus be used here
- Our previous discussion about pollution (or other negative externalities) can, conversely, be considered a situation where there is a *Public Bad*

Free-riding and crowd-out I

- Assume quasi-linear preferences and consider the condition of A 's utility max:

$$v_A'(g) = c$$

- This defines a unique level $\bar{g}$ that A prefers.

- If he can (corner solution?), A will always choose his production g_A such that the total amount of the public good becomes $\bar{g}$.

Free-riding and crowd-out II

- Best response becomes: $g_A^*(g_B) = \bar{g} - g_B$

- *Free-riding*: When B selects a high g_B, A has an incentive to lower her contribution

- (Full) *Crowd-out*: For each unit B increases his contribution, A decreases hers by the same amount (if possible); the same would happen if the government were to provide (part) of the public good

- The full crowding out (1-to-1 reduction) applies only due to quasi-linear preferences; other preferences may provide crowd-out that is greater or less than one

Lindahl

- Now we will start looking at possible solutions to the efficiency problem; The first step is the Lindahl equilibrium
- We start off by the same 2-agent model as before (recall that $c(g) = c \cdot g$)
- Assume that there is a single public entity which produces g ...

Lindahl equilibrium, the definitions

- The public good market allows each agent to buy access to any amount of public good, g_A, g_B
- In contrast to the usual markets, we will assume that a different price may be charged for each agent t_A, t_B ...
- ... but conversely we maintain that the public good (once produced) is consumed by everyone, i.e. market equilibrium here requires that all agents demand the same amount of g
- Finally, we will require the payments from the agents to cover the cost of the public good: $t_A + t_B = c$ (otherwise money will be lost)

Lindahl equilibrium, chart

Technology and Preferences	Behavior and Equilibrium
Exogenous func./var./relationships:	*The decisions of the agents:*
$u_A(x_A, g), u_B(x_B, g)$	$\max_{x_A, g_A} \quad u_A(x_A, g_A)$
c	s.t. $\quad e_A = x_A + t_A g_A$
$e_A + e_B = x_A + x_B + c \cdot g$	(Same for B)
	$\hookrightarrow$ *Conditional behavior:*
Endogenous variables:	Demand Functions
$x_A, x_B, g, \boldsymbol{g_A}, \boldsymbol{g_B}, \boldsymbol{t_A}, \boldsymbol{t_B}$	$g_A^*(t_A),\ g_B^*(t_B), x_A^*(t_A),\ x_B^*(t_B)$
	Equilibrium Conditions:
	$g_A^*(t_A^*) = g_B^*(t_B^*) = g^*$
	$t_A^* + t_B^* = c$

Lindahl equilibrium I

- What does Lindahl's equilibrium look like?

- Consider A's utility maximization problem:

$$\max_{x_A, g_A} \quad u_A(x_A, g_A)$$

$$\text{s.t.} \quad e_A = x_A + t_A g_A$$

  - Standard consumer problem with prices 1 and t_A; condition of interior solution:

$$|MRS_A| = t_A$$

Lindahl equilibrium II

- Similar for B:

$$|MRS_B| = t_B$$

- Also keep in mind the condition that the prices cover the costs:

$$t_A + t_B = c$$

- Combine:

$$|MRS_A| + |MRS_B| = c$$

The Lindahl-equilibrium is efficient

- We see that the Lindahl equilibrium is efficient!
- Analogous to the 1st welfare theorem, private goods and Walras equilibrium
  - The Walrus equilibrium (with private goods): one price per good, consumers consume different quantities
  - Lindahl equilibrium (with a public good): consumers face different prices (price discrimination) but consume the same amount of the public good

The Lindahl equilibrium is unrealistic

- We have discussed that it is unclear how we get into the (Walras / Nash) equilibrium, but Lindahl is even worse:
  - Where do the t_A and t_B prices come from? Most obvious solution: The public decides about them
  - To get the right equilibrium prices requires knowledge of the agents' willingness to pay, but there are free-rider incentives
  - If asked (directly / indirectly) the agents have a clear incentive to lie about their willingness in order to pay less

Socrative Quiz Question

True or False: If consumers have identical preferences, the Lindahl prices they have to pay will always be identical.

Other solutions, mechanism design

- Can we (the public) find other ways to choose how much g to produce?
- If we know the preferences, it's easy.
- But what if we don't? Can we get the agents to reveal their preferences in some way?
- These are questions explored within what is called *Mechanism Design*

Mechanism Design

- The public is trying to design a system (*mechanism*) where the agents must announce their preferences ...
- ... based on all agents' messages, the public decides how much of the public good is produced ...
- ... and how much each agent has to pay.

Mechanism Design, formally I

- In the context of our small 2-person economy, we can formally define a mechanism as $S_A, S_B, T_A(\cdot), T_B(\cdot), G(\cdot)$, where:
  - Each agent must send a signal $s_A \in S_A$ and $s_B \in S_B$
  - The signals determine the amount of the public good and what each agent must pay:

$$T_A(s_A, s_B), \quad T_B(s_A, s_B), \quad G(s_A, s_B)$$

  - Along with the agents' utility, the above defines a game where the agents choose their signal by maximizing:

$$U_A(s_A, s_B) = u_A\left(e_A - T_A(s_A, s_B), G(s_A, s_B)\right)$$
$$U_B(s_B, s_A) = u_B\left(e_B - T_B(s_A, s_B), G(s_A, s_B)\right)$$

Mechanism design, formally II

- We can now analyze what happens in the Nash equilibrium under different mechanisms, for example:

  1. Can we find a mechanism $S_A, S_B, T_A(\cdot), T_B(\cdot), G(\cdot)$ where the agents reveal their true preferences (willingness to pay) in the Nash equilibrium?

  2. Can we find a mechanism $S_A, S_B, T_A(\cdot), T_B(\cdot), G(\cdot)$ where the Nash equilibrium is efficient?

  3. Etc...

- We will focus mostly on 1.

A simple mechanism I

- Consider our 2-person economy and assume quasi-linear preferences as well as constant marginal cost for the public good c; a simple mechanism is:
  - The agents send a signal about how much should be produced of the public good: $S_A = S_B = \mathcal{R}$
  - The average of what the agents have said is produced and the costs are distributed proportionally according to the signals:

$$G(s_A, s_B) = \frac{s_A + s_B}{2}$$
$$T_A(s_A, s_B) = \frac{s_A}{s_A + s_B} \cdot cG(s_A, s_B)$$
$$T_B(s_A, s_B) = \frac{s_B}{s_A + s_B} \cdot cG(s_A, s_B)$$

A simple mechanism, equilibrium

- Under this mechanism, A maximizes (given s_B):

$$\begin{aligned}
&u_A\left(e_A - T_A(s_A, s_B), G(s_A, s_B)\right) \\
&= u_A\left(e_A - \frac{s_A}{s_A + s_B} \cdot cG(s_A, s_B), G(s_A, s_B)\right) \\
&= u_A\left(e_A - \frac{s_A}{2} \cdot c, \frac{s_A + s_B}{2}\right)
\end{aligned}$$

- The solution is characterized by the equation:

$$v'\left(\frac{s_A + s_B}{2}\right) = c \quad \Longleftrightarrow \quad v'(G) = c$$

- Same result as Nash equilibrium with voluntary donations.

Socrative Quiz Question

True or false: In the mechanism just outlined, if the endowment of the agents increases we get closer to the efficient solution.

Vickrey-Clarke-Groves, discrete good

- Let's look at a very famous (and elegant) mechanism: the VCG mechanism; setup:
  - Discrete public good $g = 0$ or $g = 1$ (for now) with cost c
  - N agents with quasi-linear preferences (note that the definition from earlier can easily be expanded from 2 agents to N); specifically:

$$u_i(x_i, g) = \begin{cases} x_i & \text{for} \quad g = 0 \\ x_i + v_i & \text{for} \quad g = 1 \end{cases}$$

We assume that we (the public) do not know v_i for the various agents

VCG, signals

- If the public good ends up being produced, agent i pays k_i for it; The k_is are pre-determined (taken as given by the agents) and ensure full funding: $\sum_i k_i = c$

- If there are no other payments, the net utility for i of the public good is therefore equal to:

$$n_i = u_i(x_i - k_i, 1) - u_i(x_i, 0) = v_i - k_i$$

- Each agent must send a signal $s_i \in \mathcal{R}$ that tells what their net utility n_i is (note: k_i fixed so this is mathematically equivalent to signaling v_i)

VCG, production

- The public good is produced if the sum of the reported net utilities is positive:

$$G(s_1, s_2, \ldots, s_n) = \begin{cases} 1 & \text{for} \quad S \geq 0 \\ 0 & \text{for} \quad S < 0 \end{cases} \qquad \text{where} \quad S = \sum_i s_i$$

- Before we continue, we note that this (hopefully) seems like a reasonable enough mechanism, but that there are obvious incentives to lie

- If my net utility from the public good is positive $n_i > 0$, I have an incentive to overdo my signal to ensure that it is produced ($s_i = \infty$!)

VCG, incentive for truth-telling

- The VCG mechanism solves this by introducing some payments in addition to k_i which provide incentives not to lie, so-called *Clarke taxes*

- Before introducing these, however, it is useful to define a little extra notation:
  - Let N be the true total net utility of the public good: $N = \sum_i n_i$
  - Let N_{-i} be the total true net utility, not including i: $N_{-i} = \sum_{j \neq i} n_j = N - n_i$
  - Let S_{-i} be the total reported net utility, not including i: $S_{-i} = \sum_{j \neq i} s_j = S - s_i$

VCG, Clarke taxes

- We say that agent i is *pivotal* (tipping agent) if his signal changes the decision regarding the production of the public good, ie. whose:

$$sign(S) \neq sign(S_{-i})$$

$$\text{where} \quad sign(x) = \begin{cases} 1 & \text{for} \quad x \geq 0 \\ -1 & \text{for} \quad x < 0 \end{cases}$$

- The idea of VCG is now to impose any pivotal agents a Clarke tax equal to the losses of the other agents because the decision changes:
  - If Agent i's signal makes him pivotal then he must pay the public $|S_{-1}|$ (remember S_{-1} is the reported net utility to the other agents)

VCG discrete, overall

- Overall, we can write the VCG mechanism for a discrete good as follows:

$$G(s_1, s_2, \ldots, s_n) = \begin{cases} 1 & \text{for} \quad S \geq 0 \\ 0 & \text{for} \quad S < 0 \end{cases}$$

$$T_i(s_1, s_2, \ldots, s_n) = \begin{cases} k_i & \text{for} \quad S \geq 0 \quad \text{and} \quad sign(S) = sign(S_{-i}) \\ k_i + |S_{-i}| & \text{for} \quad S \geq 0 \quad \text{and} \quad sign(S) \neq sign(S_{-i}) \\ 0 & \text{for} \quad S < 0 \quad \text{and} \quad sign(S) = sign(S_{-i}) \\ |S_{-i}| & \text{for} \quad S < 0 \quad \text{and} \quad sign(S) \neq sign(S_{-i}) \end{cases}$$

VCG reveals the preferences

- Key result: Under the VCG mechanism, it is a Nash equilibrium that everyone tells the truth ($s_i = n_i$ for all i)

- To prove this, we will consider agent i and show that $s_i = n_i$ is the best response regardless of the other's signals, ie. no matter what S_{-i} is

- We focus on $n_i \geq 0$ and then go through three possible cases ($n_i < 0$ can be dealt with using the same arguments):
  1. $S_{-i} \geq 0$
  2. $S_{-i} < 0$ and $S_{-i} + n_i \geq 0$
  3. $S_{-i} < 0$ and $S_{-i} + n_i < 0$

Case 1: $S_{-i} \geq 0$

- If I tell the truth ($s_i = n_i$) the good is bought and I am not pivotal, relative utility (relative to no project): $n_i \geq 0$

  (More detailed: $u_i(x_i - k_i, 1) - u_i(x_i, 0) = v_i - k_i = n_i$)

- If I exaggerate ($s_i > n_i$) or under-report a bit ($n_i > s_i \geq -S_{-i}$) the outcome (and utility) is the same

- If I under-report a lot ($s_i < -S_{-i}$) I become pivotal and the public good is not bought and I pay the Clarke tax, giving relative utility: $0 - |S_{-i}| < 0$

- Bottom line: It is a best response to tell the truth ($s_i = n_i$)

Case 2: $S_{-i} < 0$ and $S_{-i} + n_i \geq 0$

- If I tell the truth ($s_i = n_i$) I am pivotal, the good is purchased and I pay the Clarke tax, giving relative utility: $n_i - |S_{-i}| \geq 0$

- If I exaggerate ($s_i > n_i$) or under-report slightly ($n_i > s_i \geq -S_{-i}$) the outcome (and utility) is the same

- If I under-report a lot I am not pivotal and the good is not produced, giving relative utility: 0

- Bottom line: It is a best response to tell the truth ($s_i = n_i$)

Case 3: $S_{-i} < 0$ and $S_{-i} + n_i < 0$

- If I tell the truth ($s_i = n_i$) I am not pivotal, the good is not produced, giving relative utility: 0

- If I under-report ($s_i < n_i$) or exaggerate a bit ($s_i < -S_{-i}$) the outcome (and utility) are the same

- If I exaggerate a lot I am pivotal, the good is produced and I pay the Clarke tax, giving relative utility: $n_i - |S_{-i}| < 0$

  (Here we use: $S_{-i} + n_i < 0 \iff n_i < -S_{-i} \Rightarrow n_i < |S_{-i}|$)

- Bottom line: It is a best response to tell the truth ($s_i = n_i$)

VCG, pros

- We have now proven that it is a Nash equilibrium to tell the truth under the VCG mechanism (check the three cases with $n_i < 0$ yourselves)
- The VCG mechanism and its use of Clarke taxes create an incentive to tell the truth...
- ... hereby “the public” can figure out the preferences of individuals (and whether the public good is subsequently produced)

VCG, cons

- VCG (and our derivations) require quasi-linear preferences (otherwise the Clarke tax gives rise to income effects)

- The mechanism's outcome is *not efficient* because Clarke taxes are wasted

  - Note that the Clark taxes are of no use to anyone!
  - The mechanism does not work if "the public" uses the tax money for the benefit of the agents ...
  - ... because then there will be incentives to lie in order to increase the other agents' Clarke taxes

- Also note that the mechanism will typically make someone worse off (n_i will be negative for someone)

Socrative Quiz Question

True or false: The VCG mechanism can be interpreted as follows: “The payment each player has to make is equal to the opportunity cost that occur due to his or her participation.”

VCG, continuous case

- VCG works in the same way with continuous public goods (see Nechyba):
  1. The agents send a signal indicating their utility / demand curve
  2. The public calculates and implements the optimal level of the public good based on the signals
  3. The public also calculates the optimal level that would apply if each of the agents' signal was not accounted for
  4. Each agent now pays an amount consisting of two elements: a fixed unit price of good k_i covering the cost ...
  5. ... and an amount (Clarke tax) based on how the agent's signal changes the consumer surplus for the other agents (from 2 and 3)

What have we learned?

- Public goods create free-riding problems
- Calculation of Lindahl equilibria
- The Vickrey-Clarke-Groves mechanism and its properties