Mikro II, lecture 10a

Introduction to Public Goods

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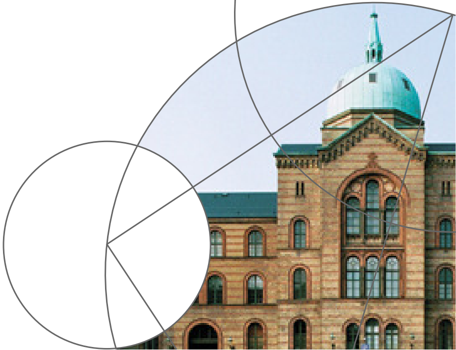


Plan for the lecture

1. The efficient level of public goods
2. Equilibrium under private production of public goods

Public goods, definition

- Public goods are *non-rival* and *non-excludable*:
  - The fact that a person consumes the good does not affect the ability of others to do the same
  - No one can be prevented from consuming the good
- Means that if a person consumes the amount of g then everyone else can

Public goods, model I

- Edgeworth economy with two consumers: A and B;
- Consume two (continuous) goods: a (private) good (money / consumption), x_A, x_B
  and a (public) good, g_A, g_B
- Utility Functions: $u_A(x_A, g_A)$, $u_B(x_B, g_B)$
- Endowment only of the first (private) good: $(e_A, 0)$ and $(e_B, 0)$

Public goods, model II

- Now we explicitly state that the public good is *public*:

$$g_1 = g_2 = g$$

- In addition, suppose that the public good can be purchased (produced) using the private good via the cost function $c(g)$, giving the following set of possible quantities:

$$e_A + e_B = x_A + x_B + c(g)$$

The model until now

Technology and Preferences	**Behavior and Equilibrium**
Exogenous func./var./relationship: $u_A(x_A, g), u_B(x_B, g)$ c $e_A + e_B = x_A + x_B + c(g)$ *Endogenous variables:* x_A, x_B, g	*The decisions of the agents:* $\hookrightarrow$ *Conditional behavior:* *Equilibrium Conditions:*

Efficient states I

- We will now characterize the efficient states
- Not quite as simple in this model as in others
- We therefore use a (slightly) new method:
  - Maximize the utility of one consumer under the condition that the other gets a certain utility level and that the condition is possible

Efficient states II

$$
\begin{aligned}
\max_{x_A, x_B, g} \quad & u_A(x_A, g) \\
\text{s.t.} \quad & u_B(x_B, g) \geq \bar{u} \\
& e_A + e_B = x_A + x_B + c(g)
\end{aligned}
$$

- When discussing Principal Agent models we argued that any solution to this kind of problem must be efficient
- A little further argumentation shows that any efficient state must be the solution to the above problem for some $\bar{u}$:
  1. Considering any inefficient condition, B must have some utility level u'
  2. The state is efficient$\longrightarrow$ it cannot be possible to make A better off $\longrightarrow$ state solves the above problem for $\bar{u} = u'$

Efficient states III

- Solve the problem with Lagrange

$$L(x_A, x_B, g) = u_A(x_A, g) - \lambda_B\left(u_B(x_B, g) - \bar{u}\right) - \lambda_G\left(x_A + x_B + c(g) - e_A - e_B\right)$$

- FOCs:

$$\begin{aligned} \frac{\partial u_A}{\partial x_A} - \lambda_G &= 0 \\ -\lambda_B \frac{\partial u_B}{\partial x_B} - \lambda_G &= 0 \\ \frac{\partial u_A}{\partial g} - \lambda_B \frac{\partial u_B}{\partial g} - \lambda_G c'(g) &= 0 \end{aligned}$$

Efficient states IV

- Rewrite the FOCs:

$$\frac{\partial u_A}{\partial x_A} = \lambda_G$$

$$-\lambda_B \frac{\partial u_B}{\partial x_B} = \lambda_G$$

$$\frac{1}{\lambda_G}\frac{\partial u_A}{\partial g} - \frac{\lambda_B}{\lambda_G}\frac{\partial u_B}{\partial g} = c'(g)$$

- Insert for λ_G in the last equation:

$$\frac{\frac{\partial u_A}{\partial g}}{\frac{\partial u_A}{\partial x_A}} + \frac{\frac{\partial u_B}{\partial g}}{\frac{\partial u_B}{\partial x_B}} = c'(g)$$

Efficient states V

- Remember definition of (numeric) MRS:

$$|MRS_A(x_A, g)| + |MRS_B(x_B, g)| = c'(g)$$

- Intuition: MRS is willingness to pay for a unit of public good wrt. item 1 (money)

- If the consumer is willing to pay more than the marginal cost for an extra unit of the public good, then more of the public good should be supplied (and vice versa)

Special case: Quasi-linear preferences.

- An important special case is quasi-linear preferences:
  $u_A(x_A, g) = v_A(g) + x_A,\ u_B(x_B, g) = v_B(g) + x_B$

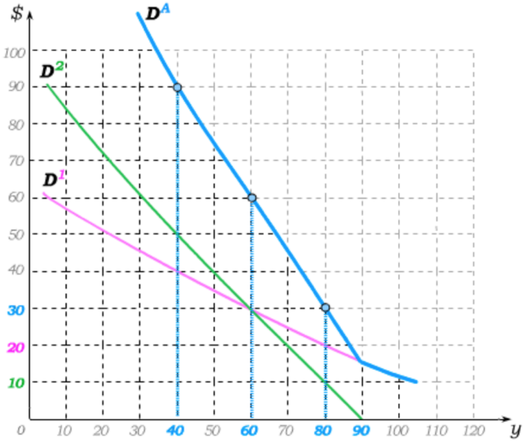
- The condition for efficiency will then become (check yourself)

$$v'_A(g) + v'_B(g) = c'(g)$$

- Note here that x_A and x_B are not included: for quasi-linear preferences, the efficient g is independent of the distribution of other goods (no income effects)

  (Note: The above results ignore corner solutions)

Graphically



- Remember: the demand curve is the willingness to pay; quasi-linear pref.: $p_A(g) = v'_A(g), p_B(g) = v'_B(g)$

- Remember for private goods: total demand (willingness to pay) is found by *horizontal* addition

- Everyone can use public goods at the same time: total willingness to pay can be found by *vertical* addition

Discrete public good

- Some public goods are discrete: $g = 1$ or $g = 0$

- Here it is useful to define willingness to pay for the good r_A, r_B and realize that with quasi-linear preferences it is independent of other goods:

$$u_A(x_A - r_A, 1) = u_A(x_A, 0) \iff r_A = v_A(1) - v_A(0)$$
$$u_B(x_B - r_B, 1) = u_B(x_B, 0) \iff r_B = v_B(1) - v_B(0)$$

- In that case, it is efficient to produce the public good $g = 1$ if and only if:

$$r_A + r_B \geq c(1) - c(0)$$

(without quasi-linear pref. it gets a little more complicated because willingness to pay depends on other goods (income effects))

Private production

- Let us return to the continuous case and examine an equilibrium where the agents themselves provide for the public good (voluntary donation)
- To simplify, we assume constant price / cost of the public goods: $c(g) = c \cdot g$
- Assumption: Every agent can provide some of the public good, g_A, g_B, thus $g = g_A + g_B$
- Nash Equilibrium: The agents maximize given the other's decision. For A we have:

$$\max_{g_A} \quad u_A(x_A, g_A + g_B)$$
$$\text{s.t.} \quad e_A = x_A + c \cdot g_A$$

Private production of public good (donations)

Technology and Preferences	**Behavior and Equilibrium**
Exogenous func./var./relationships:	*The decisions of the agents:*
$u_A(x_A, g), u_B(x_B, g)$	$\max_{g_A} \quad u_A(x_A, g_A + g_B)$
c	s.t. $\quad e_A = x_A + c \cdot g_A$
$e_A + e_B = x_A + x_B + c(g)$	(same for B)
$g = g_A + g_B$	
	$\hookrightarrow$ *Conditional behavior:*
Endogenous variables:	Best response
x_A, x_B, g, g_A, g_B	$g_A^*(g_B)$, $g_B^*(g_A)$
	Equilibrium Conditions:
	Nash equilibrium
	$\bar{g}_A = g_A^*(\bar{g}_B)$, $\bar{g}_B = g_B^*(\bar{g}_A)$

Socrative Quiz Question

Which problem we have previously seen is this closest to?

- Consumer tax on the good.
- Negative externality.
- Asymmetric information about good quality.
- Positive externality.

Solution

- Insert for x_A in the utility: $u_A(e_A - c \cdot g_A, g_A + g_B)$ and take the first-order condition wrt. g_A:

$$-c \cdot \frac{\partial u_A}{\partial x_A} + \frac{\partial u_A}{\partial g} = 0 \iff \frac{\frac{\partial u_A}{\partial g}}{\frac{\partial u_A}{\partial x_A}} = c$$

- The condition for B is the same; write up using MRS:

$$|MRS_A| = c$$
$$|MRS_B| = c$$

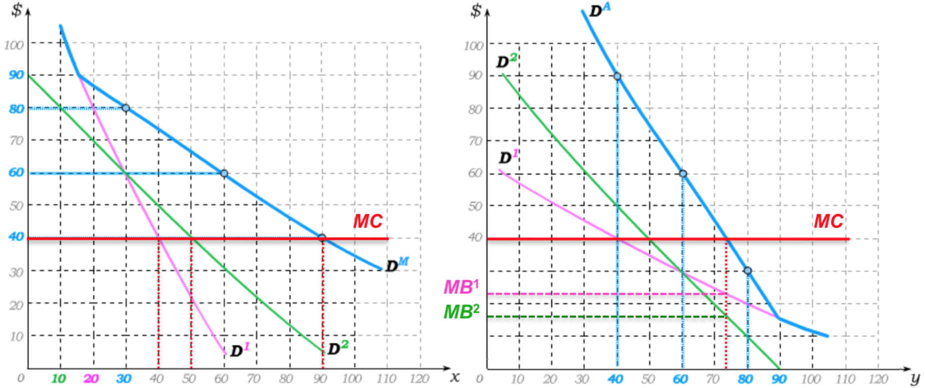
- Compare with efficiency condition:

$$|MRS_A| + |MRS_B| = c$$

Discussion

- We see that the equilibrium is generally not efficient.
- When agents make individual decisions, they only consider their own willingness to pay.
- Intuitively obvious that the equilibrium will have too little public goods (can be formally shown by MRS decreasing in g).
- In the special case of quasi-linear pref. we get: $v'_A(g) = c, \quad v'_B(g) = c$

Graphical equilibrium



- Private equilibrium solves $v_A'(g) = v_B'(g) = c$, efficiency requires $v_A'(g) + v_B'(g) = c$

Socrative Quiz Question

True or false: Imagine that the curvature of the cost curve is such that there are economies of scale to the production of the public good. In this case, the market quantity with voluntary contributions is closer to the efficient quantity.

What have we learned?

- Calculating the efficient amount of a discrete or continuous public good
- Calculation of an equilibrium with private production of the public good
- How the private equilibrium looks compared to the efficient level