Question 1) 2020 sommer

a) What is the maximization problem of Carlsborg as a function of QC and QT ?

The total produciton from the two firms is $Q = Q_C + Q_T$

Sets up the profit max problem

$$\max_{Q_C,Q_T} \pi_C = p(Q_C, Q_T) \cdot Q_C - C_C(Q_C)$$

The inverse demand function is $p(Q) = 110 - \frac{1}{10}Q$. Finding Carlsborg best response function by optimizing profit for Carlsborg on Tuborg

$$\pi_C = Q_C(101 - \frac{1}{10}(Q_C + Q_T)) - Q_C \Leftrightarrow$$

$$\pi_C = (101 - \frac{1}{10}(Q_C + Q_T) - 1)Q_C$$

b) Which quantity of beer will the two firms produce together in equilibrium?

Finding the best response function:

$$\max_{Q_C,Q_T} \pi_i = p(Q_C, Q_T) \cdot Q_I - C(Q_C + Q_T)$$

$$\pi_i = Q_i(101 - \frac{1}{10}Q_i) - Q_j$$

$$\pi_i = (101 - \frac{1}{10}(Q_C + Q_T) - 1)Q_C$$

$$\frac{\partial \pi_i}{\partial Q_C} = 100 - \frac{1}{5}Q_C - \frac{1}{10}Q_T = 0 \Leftrightarrow$$

$$\frac{-2Q_C - Q_T}{10} = 0 \Leftrightarrow Q_C = 500 - 0.5Q_T$$

Because they have same production opportunity we say:

$$Q_i = 500 - 0.5Q_j$$

where i=C,T and $i \neq j$, for $j = C, T$

Inserting this into Q_C gives;

$$Q_j = 500 - 0.5(500 - 0.5Q_T) \Leftrightarrow Q_T = 0.25Q_T + 250 \Leftrightarrow$$

$$Q_T = 333.33$$

Because $Q_T = Q_C$ they each produce 333.33 liters and have a market quantity 666.66 liters.

c) Assume that the firms are instead competing in prices ‘a la Bertrand. How many liters will the two firms produce together in equilibrium?

The two firms will under-cut each other intil the price is equal to marginal cost giving a price of 1. They will not go lower than that because otherwise they would make negative profits, for higher prices a firm would have incentive to undercut the other firm

Inserting into the inverse demand function, giving a total quantity of;

$$1 = 101 - \frac{1}{10}Q \Leftrightarrow Q = 1000$$

d) Go back to the initial situation and assume that Tuberg and Carlsborg merge. Which quantity of beer will the resulting firm produce?

$$P = 101 - \frac{1}{10}Q$$

$$TR = p(Q) \cdot Q = 101Q - \frac{1}{10}Q^2$$

$$MR = MC \Leftrightarrow$$

$$101 - \frac{1}{5}Q = 1$$

$$Q = 500$$

They will produce 500 when merge

e) Will the resulting firm in d) have higher, lower or the same profits as the combined profits of Tuberg and Carlsborg in the initial situation? Explain the economic mechanism why profits are different or the same. There is no need to use calculations in your response.

The resulting firm will have higher profits than the combined profits of Tuberg and Carlsborg in the initial situation. The quantities that would maximize the joint profit of the two firms in the initial situation would be half of the monopoly quantity for each firm. However, this cannot be an equilibrium under Cournot competition, as each firm would have an incentive to make a one-sided deviation and increase profits by producing more. The merger of the two firms makes them a monopolist that makes decisions as one entity, and therefore can maximize the joint profits of the two firms. Alternatively, one could argue that under Cournot competition, firms do not fully incur (internalize) the cost of an increase in their produced quantity in the form of having a lower price (the "cost" of lowering the price occurs to both firms). One could also argue that, since the joint quantity is different when the two firms make decisions together than when they make decisions separately, the combined profit has to be higher, as they might as well have continued producing the old quantity. One could also argue that the Cournot case entails some competition, which brings the equilibrium outcome closer to perfect competition, which features zero profits.

f) Go back to the initial situation and assume that Tuberg now faces constant marginal cost of 3, while Carlsberg continues to produce at marginal cost of 1. Which quantities would the two firms produce in equilibrium?

Tuborgs best response function becomes:

$$\max_{Q_C, Q_T} \pi_T = p(Q_C, Q_T) \cdot Q_I - C(Q_T)$$

$$\pi_T = (101 - \frac{1}{10}(Q_C + Q_T))Q_T - C(Q_T)$$

$$\pi_T = 101Q_T - \frac{1}{10}(Q_C + Q_T)Q_T - C(Q_T) \Leftrightarrow$$

$$\frac{\partial \pi_T}{\partial Q_t} = 101 - \frac{2Q_t + Q_C}{10} - 3 = 0 \Leftrightarrow$$

$$Q_T = \frac{980 - Q_C}{2} = Q_T = 490 - 0.5Q_C$$

Carlsborg has the same response function.

$$Q_C = 500 - 0.5 * (Q_T)$$

Intersting into each other gives:

$$Q_C = 500 - 0.5 * (490 + 0.5Q_C) \Leftrightarrow Q_C = 340$$

$$Q_T = 490 - 0.5(340) = 320$$

g) What will happen under Bertrand competition if Tuberg faces constant marginal cost of 3, while Carlsberg continues to face marginal cost of 1? Explain in words whether the overall quantity produced in the market and the price will be different than under c), and – if yes – in which directions they will change relative to c), and why. It is not necessary to calculate the new market price and quantity.

Under Bertrand competition, Tuberg will no longer want to offer beer at a price of 1, as it will make negative profits in this case, so it has to raise its price to 3. Carlsberg will play a best response to that, which means it will undercut Tuberg by a tiny amount ε and capture the entire market. The overall market price will increase, and the overall market quantity will decrease relative to the situation under c).

Question 2) 2020 sommer

$$Q_C = 500 - 0.5Q_T$$

Suppose you are a member of the successful newcomer band "The Public Good Providers". "The Public Good Providers" have three fans, Claus, Søren and Niels, who have already purchased tickets for a concert of your band. Assume that there are no other guests at the concert. If your band practices, this increases the quality Q of the concert. Claus, Søren and Niels consider paying your band for practicing, and your band has offered them to practice as many hours as they want in return for receiving one kroner per hour. Music quality will be 0 without practice and increase by one unit with each hour of practice. However, while Søren places great importance on the music being of high quality, Niels places not so much importance on it, and Claus does not care at all about the quality of the music and only goes to the concert to hang out with the others. They have the following utility functions, where xC , xS and xN are the amounts of money Claus, Søren and Niels "consume" in the end, respectively:

$$u_C(x_C) = x_C$$

$$u_S(x_S, Q) = x_S + 4\sqrt{Q}$$

$$u_N(x_N, Q) = x_N + 2\sqrt{Q}$$

a) What levels of the public good of music quality QC , QS , QN will each of the three contribute if they make decisions individually and take the other players' decisions as given? What is the total amount provided?

Q is the quality of the concert. Each of the four fans can raise the total quality given by, $Q = Q^C + Q^S + Q^N$

Each fan can use his own 100 kr. to raise the quality Q^i or don't so they can use the money later.

We have a cost to raise the quality: $C(Q) = 1 * Q$.

When the agent has to contribute to the public good, they will choose $|MRS_i| = C'(Q)$. Because $C'(Q) = 1 \rightarrow MC = 1$

Find find MRS and solve for the three agents:

Claus:

$$|MRS_C| = |\frac{\frac{\partial u_C}{\partial Q}}{\frac{\partial u_C}{\partial x_C}}| = \frac{0}{1} = 0$$

We see that Claus does not want to raise the music quality, $Q^C = 0$

Søren:

$$|MRS_S| = |\frac{\frac{\partial u_s}{\partial Q}}{\frac{\partial u_s}{\partial x_s}}| = \frac{2Q^{-1/2}}{1} = \frac{2}{\sqrt{Q}}$$

$$\frac{2}{\sqrt{Q}} = 1 \Leftrightarrow$$

$$Q = 4$$

Niels:

$$|MRS_N| = |\frac{\frac{\partial u_N}{\partial Q}}{\frac{\partial u_N}{\partial x_N}}| = \frac{Q^{-1/2}}{1} = \frac{1}{\sqrt{Q}}$$

$$\frac{1}{\sqrt{Q}} = 1 \Leftrightarrow Q = 1$$

Claus chooses Q=0 no matter what.

Niels whishes to, $Q = Q^C + Q^S + Q^N = 1 \Leftrightarrow 1 = Q^N + Q^S$. If Niels chooses to pay 1 will he be satisfied.

Søren whishes to, $4 = Q^C + Q^N + Q^S$ but because Niels pays 1 and Claus pays 0, can Claus choose to pay, $4 = Q^S + 0 + 1 \Leftrightarrow 4 = Q^S$

Niels sees that Søren pays 3 and now he does not need to pay because Søren fulfill Niels whishes. Therfore Niel will pay $Q^N = 0'$. Søren realises that Niels does not want to pay and ends up paying $4 = Q^C + Q^N + Q^S \Leftrightarrow 4 = 0 + 0 + Q^S$

We end up with

$$(Q^C, Q^N, Q^S) = (0, 0, 4)$$

this this our NE. There will be bought 4 hoours of practice in equilibrium

b) What is the socially optimal level of music quality?

The social optimum level is found wby solving:

$$|MRS_C| + |MRS_S| + |MRS_N| = C'(Q)$$

We use again that MC=1:

$$|MRS_C| + |MRS_S| + |MRS_N| = MC \Leftrightarrow$$

$$0 + \frac{2}{\sqrt{Q}} + \frac{1}{\sqrt{Q}} = 1 \Leftrightarrow$$

$$Q^* = 9$$

c) Your band understands that Claus, Niels and Søren will buy less music quality than the socially optimal level. Your band therefore decides to try to overcome this problem by letting them pay Lindahl prices. What are the Lindahl prices t C , t S , t N that each of the three fans would have to pay in the Lindahl equilibrium?

The Lindahl-prices is, $t_i = |MRS_i|$ and we now the social optimum is Q=9

$$t_C = |MRS_C| = 0$$

$$t_S = \frac{2}{\sqrt{9}} = 2/3$$

$$t_N = |MRS_N| = \frac{1}{\sqrt{9}} = 1/3$$

d) Why may Claus, Niels and Søren be unwilling to truthfully provide the information you need to calculate the socially optimal level of the public good and the Lindahl prices? Explain in words.

There is risk for free-riding. THe thre agents nows that their Lindahl-prices is dependen on their MRS in social optimum $Q^* = 9$. They can have incentive to lie about their MRS so they have to pay less to use the public good and thereby free-ride because the other agents will pay more.

e) What are Claus', Niels' and Søren's net utilities, nC, nN and nS, for going from music quality of 0 to music quality of 16 if each of them has to pay 5 kroner for the provision of the public good?

The agents can raise Q, so the quality raises from $Q = 0$ to $Q = 16$. If they do not rause the quality, $x_i = 100 - 0$. If they raise the quality for 5 kr. they will end up with 5 kr. less: $x_i = 100 - 5 = 95$

The netto-utility from buying practice hours is the differens between utility when Q=0 and Q=16: $n_i = u_i(x_i = 95, Q = 16) - u_i(x_i = 100, Q = 0)$

$$n_C = u_C(x_C = 95, Q = 16) - u_C(x_C = 100, Q = 0)$$

$$n_C = 95 - 100 = -5$$

$$n_S = 95 + 4\sqrt{16} - 10 - 4\sqrt{0} = 11$$

$$n_N = 95 + 2\sqrt{16} - 100 - 2\sqrt{0} = 3$$

f) Assume that the Vickrey-Clarke-Groves mechanism is successful in the sense that all three report their true net utilities. Will the public good be purchased? Which agent is pivotal? What is the Clarke tax that this agent will have to pay?

The sum of the netto-utility in soceity is:

$$N = \sum n_i = -5 + 11 + 3 = 9 > 0$$

It is optimal if the public good is bought if all speek the truth. We see that if Claus has utility 0, the total netto-utility will be:

$$N_{-C} = 11 + 3 = 13 > 0$$

There wil still be bought public goods. Claus can't change the decision and is not a pivotal agent.

If Sørens netto-utility is 0, the total utility will be:

$$N_{-S} = -5 + 3 = -2$$

The public good will not be bought. Søren is a pivotal agent.

Niels is not a pivotal agent bevause i can't change the decision. His netto utility is not higher than the total netto utility.

We can put a Clarke-tax on our pivotal agent. The tax must equal the loss the society suffers from when Søren lies. Søren must pay a Clark-tax=2.

Question 3) 2020 sommer

a) Mette suggests to apply the Democracy Social Choice Function (SCF) and find the optimal decision through pairwise voting. That is, the friends will vote between two movies, and the winning movie goes on to the next round, where there is a vote between that movie and another option which it has not yet won over. The process is repeated until there is an option that has won over all the other options. Explain what problem occurs in this process and how it arises.

The suggested procedure will lead to a Condorcet cycle, meaning that the proposed decision rule may lead to intransitive social preferences even if all individual preferences are transitive (and total). To see this, consider the comparison Star Wars against Fast and Furious. Fast and Furious will win, as Asger and Jeanet prefer it to Star Wars. Now Fast and Furious is compared to Transformers. Transformers will win as it is preferred by Mette and Jeanet. Now Transformers is compared to Star Wars. Star Wars will win, as Mette and Asger prefer it, and we are back at a point where we would have to compare Star Wars to Fast and Furious. We do not get a Condorcet winner. We could also arrive at this result by doing the votes in different order

b) Jeanet suggests to over-come this problem by forbidding voting on options that have already lost in a vote, and that she decides the order of the voting. The other two agree. Which pair of movies is Jeanet going to suggest for the first round of voting? Is she going to succeed in getting to watch her favorite movie in case all three vote according to their preferences? Explain what new issue occurs due to the change in the voting rules.

Jeanet will suggest that they first vote on Star Wars vs Fast and Furious. Fast and Furious will win and they will vote on Fast and Furious against Transformers. Transformers will win and the voting 5 will be over, as Star Wars has already lost a vote. In this way Jeanet will succeed in watching her first choice. This illustrates that the Democracy SCF with pairwise voting and forbidding votes on options that have already lost overcomes the problem of Condorcet cycles. However, this comes at the cost of creating a new problem, namely giving "Agenda Setting Power" to the agent deciding about the order in which votes over different options occur.

c) Asger thinks a step ahead and notices that, despite the fact that the voting is done in the order suggested by Jeanet, he can improve the outcome for himself through smart voting behavior. Which movie does he have to vote for (against his preferences) and in which voting round does he have to do so in order to improve his situation? Which movie would the three friends watch in that case? Assume that the other two will vote in accordance with their true preferences and that Asger knows this

Asger prefers both Star Wars and Fast and Furious to Transformers. If Asger votes for Star Wars in the first round even though he prefers Fast and Furious, the second round will be a vote of Star Wars against Transformers, which is preferred by Mette and Asger, so Star Wars wins, and Asger ends up with his second choice instead of his third choice.

d) Can we apply the median voter theorem in the situation described under a)? Why or why not?

The median voter theorem tells us that if preferences are rational (total and transitive) and singlepeaked, then the ideal point of the median voter is socially optimal and will be a Condorcet winner. In the situation under a) we do not have a Condorcet winner – we get a Condorcet cycle. The problem is that the individual preferences are not single-peaked (even though they are rational). This is because there is no objective numeric ordering of the different options, such that we can say "the ideal point of Mette is below the ideal point of Asger" or similarly. This implies that we cannot identify a median voter.

Question 4

a) "Franchising contracts will typically involve no information rents to the agent, i.e. the agent will typically earn her reservation utility. This tends to be the case regardless of whether the agent is risk-neutral or risk-averse, and of whether the revenue is risky or not."

This statement is true. In principal agent models where the effort of the agent cannot be observed after the contract has been signed, one way to induce positive effort of the agent is franchising. Franchising means that the principal allows the agent to produce and keep the profits to herself, but the agent in turn has to pay a fixed amount to the principal. The worker gets the full marginal return of her work and is therefore "residual claimant". This will ensure that the agent chooses the level of effort optimally. The principal will then charge the agent the highest amount such that the agent still accepts the offer, which means that the agent will earn her reservation utility in expectation. Otherwise the principal could increase her profits by charging a higher price and the agent would still accept. This will be true no matter whether the agent is risk-averse or not, and no matter whether there is a random component driving the revenue or not. If the agent is risk-averse and there is a random component, however, the agent will have a lower willingness to pay for the franchising contract. This will lower the principal's expected profits, so there will be information cost to the principal. However, the agent will still earn her reservation utility in expectation, and therefore not earn any information rents here.

b) "In the grand scheme of things, allowing firms to take out patents for new products they have invented through research and innovation is a bad thing since it leads to market power." (Note: A patent guarantees a firm to be the only seller of a new product, typically for a duration of a few years.)

On the one hand, allowing one firm to be the sole seller of a particular product indeed leads to welfare losses compared to a case where this product is offered by many firms, since this firm will likely behave as monopolist and charge a price that is too high and produce a suboptimally low quantity. However, there will most likely still be a welfare improvement from having one monopolist selling the new product compared to a situation where the product is not produced at all. Patents will provide an incentive to firms to invest in research and innovation and develop new products. To see this, note that investing in research and innovation creates positive externalities for other firms if those other firms are allowed to also produce and sell the newly invented products. This means that every individual firm will have an incentive to underinvest in research and innovation. Allowing firms to take out patents removes those positive externalities to other firms, increasing each firm's incentive to invest in research and innovation. One could invoke the Coase theorem and argue that patents assign property rights to a good whose property rights were previously undefined, and could therefore improve efficiency. One could also argue that without earning positive profits as in the monopoly case, firms would not be able to pay for the research and innovation, so they may not do it. In practice, one could come up with policies that limit patents for a certain duration, or one could regulate the new monopolies in various ways to make sure that welfare losses are minimized.

Question 1) 2020 sommer RE

Demand for steel is:

$$D(p) = \max\{800 - 2p, 0\}$$

Production technology is:

$$C(y) = 300 + 0.25y^2$$

a) What are the equilibrium price and quantity in the market for steel?

Find the inversen demand function:

$$p^* = 400 - \frac{y}{2}$$

Finding MC:

$$MC(y) = \frac{\partial C(y)}{\partial y} = 0.5y$$

Sets MC equal to inverse demand:

$$400 - \frac{y}{2} = 0.5y \Leftrightarrow y^* = 400$$

Inserting in the price function:

$$p^* = 400 - \frac{400}{2} = 200$$

b) What are the consumer surplus and the producer surplus in the market for steel?

$$CS = \frac{1}{2}(800 - 400) \cdot 200 = 40000$$

$$PS = \frac{1}{2}(200 - 0) \cdot 400 + (200 - 200) \cdot 400 = 40000$$

Now assume that the production of steel causes pollution. The cost that occurs to society from pollution, e(y), can be described by the following function:

$$e(y) = 100y + 0.25y^2$$

c) What is the socially optimal level of steel production?

Sum the private cost and the cost from pollution to get the total social cost arising from the production of steel as

$$SC(y) = e(y) + C(y) = 100y + 0.25y^2 + 300 + 0.25y^2 = 0.5y^2 + 100y + 300$$

Taking the derivative w.r.t y give the total social marginal cost for the production of steel as:

$$SMC(y) = \frac{\partial SC(y)}{\partial y} = y + 100$$

sets

$$p^* = SMC \Leftrightarrow$$

$$400 - \frac{y}{2} = y + 100 \Leftrightarrow y_S^* = 200$$

d) What is the total net surplus in the market for steel, if you take the additional cost created through pollution into account? Explain the consequences of your results in words.

Finding the ares between the total SMC and MC:

$$TC_{pol} = e(y^*) \Leftrightarrow 100 * 400 + 0.25 * 400^2 = 80000$$

$$N = PS + CS - TC\text{pol} =$$

$$40000 + 40000 - 80000 = 0$$

This implies that closing down the market for steel or forbidding steel production would not reduce welfare.

e) Is there a deadweight loss arising from pollution? Explain why it arises or not

Finding TS under efficient allocation, will be given by the area between the inverse demand curve and the total social marginal cost curve.

$$TS = \frac{1}{2}(300 * 200) = 30.000$$

$$DWL = 30.000 - 0 = 30.000$$

The deadweight loss arises because the firm does not take the costs occurring to society in the production of steel into account when making decisions.

f) The government wants to introduce a Pigouvian tax to achieve the socially optimal level of steel production. The government proposes that the firm has to pay a constant tax on each unit of steel produced. How high would the Pigouvian tax have to be to achieve the socially optimal quantity?

) The tax has to be equal to the difference between private marginal cost and total social marginal cost at the efficient level. This will be given by t=$(SMC(y_S^*) - MC(y_S^*)) = 100 + 200 - 0.5 * 200 = 200$.

Thus, a Pigouvian tax of 200 on the production of steel will ensure production of the socially optimal quantity in the market for steel

g) Would you recommend the government to introduce the Pigouvian tax or rather to leave the market unregulated?

We compare total net surplus with and without the tax.

$$CS = \frac{1}{2}(y^* - SMC(y_S^*)) \cdot y_S^* = 0.5(400 - 300) * 200 = 10000$$

$$PS = \frac{1}{2}(MC(y_S^*) - FC^*) \cdot y_S^* = \frac{1}{2}(100 * 200) = 10000.$$

$$Tax = t \cdot y_S^* = 200 * 200 = 40000$$

The additional social cost from the externality is the pollution cost of steel in the new equilibrium, which is given by

$$E = e(y_S^*) \Leftrightarrow 100 * 200 + 0.25 * 200^2 = 30,000$$

Net surplus is:

$$TS_{new} = CS + PS + Tax - E = 10000 + 10000 + 40000 - 30000 = 30000$$

This is higher than 0 derived under (d) so we can recommend the tax.

Question 2) 2020 sommer RE

a) Write down the individual rationality (IR) constraint that makes sure that Asger works for you rather than taking his outside option, and solve it for wH

IR-betingelsen skal sikre, at nytten ved at arbejde er mindst lige så stor, som outside option. Da principalen ønsker, at agenten, Asger, arbejder e = 1, tager vi udgangspunkt i dette.

Hvis Asger arbejder med e = 1, vil ssh. for at sælge en cykel være 0.9, mens ssh. for ikke at sælge en cykel er 1 − 0.9 = 0.1.

Outside option is $\bar{u} = 0$

$$\text{Exp. utility to work e=1} \geq \text{outside option} \Leftrightarrow$$

$$0.9 \cdot u(e = 1, w_H) + 0.1 \cdot u(e = 1, w_L) \geq 0 \Leftrightarrow$$

$$0.9 \cdot (w_H - 90 * 1) + 0.1(w_L - 90 \cdot 1) \geq 0 \Leftrightarrow$$

$$0.9w_H + 0.1w_L - 90 \geq 0 \tag{IR}$$

We use that $w_L = 0$

$$0.9w_H - 90 \geq 0 \Leftrightarrow$$

$$w_H \geq 100 \tag{IR'}$$

b) Write down the incentive compatibility (IC) constraint that makes sure that Asger puts in a high level of effort rather than a low level, and solve it for wH.

$$\text{Exp. utility from working e=1}\geq\text{exp. utility from working e=0} \Leftrightarrow$$

$$0.9 \cdot u(e = 1, w_H) + 0.1 \cdot u(e = 1, w_L) \geq 0.5 \cdot u(e = 0, w_H) + 0.5 \cdot u(e = 0, w_L) \Leftrightarrow$$

$$0.9 \cdot (w_H - 90 * 1) + 0.1(w_L - 90 \cdot 1) \geq 0.5 \cdot (w_H - 90 * 0) + 0.5(w_L - 90 \cdot 0) \Leftrightarrow$$

$$0.9w_H + 0.1w_L - 90 \geq 0.5w_H + 0.5w_L \tag{IC}$$

Uses that $w_L = 0$

$$0.9w_H + 0.1 * 0 - 90 \geq 0.5w_H + 0.5 * 0$$

$$w_H \geq 225 \tag{IC'}$$

c) Set up formally your (the principal's) profit maximization problem including constraints, assuming that you want to induce high effort. What wage will you pay Asger? What are your expected profits and what is Asger's expected utility? Note: What you maximize is your expected hourly profit.

We knoe that the revenue is 1.000 kr. for each sold bike.

We assume that the wage contract above assures that Asger works e=1. There is 0.9 chance for Asger to sell 1 bike in a hour. We have to then pay w_H

There is also a canche on 0.1 for Asger to sell 0 bikes and we only have to pay him w_L. The profit is:

$$\Pi = 0.9 * (1000 - w_H) + 0.1(0 - w_L)$$

The principal will maximixe the profit, but (IR') and (IC') must be true. We use $w_L = 0$

$$\max_{w_H} 0.9 * (1000 - w_H) = 900 - 0.9 w_H \text{u.b.b}$$

$$w_H \geq 100 \tag{IR'}$$

$$w_H \geq 225 \tag{IC'}$$

The principal wil offer $w_H = 225$ so both condition is true.

Asger gets following utility:

$$e = 1 = 0.9 \cdot (225 - 90 * 1) = 112.5$$

The principals profit is:

$$\Pi = 0.9 * (1000 - 225) = 697.5$$

d) We have assumed that you (the principal) want to induce high effort by Asger. Is this optimal for you?

We can secure he works e=0 and not e=1. The wage is equal to $w_H = w_L = 0$. When the agent works with low effort the chance for selling a bike is 0.5 and 0.5 chance for not selling a bike.

The profit is:

$$\Pi = 0.5(1000 - w_H) + 0.5 * (0 - w_L) = 0.5 * 1000 = 500$$

The principal gets a lower profit and will not have a worker with low work effort.

e) Would your answer to d) change if we instead assumed pL = 0.3? Why or why not?

If the worker works with e=0 the chance is 0.3 for selling a bike while having a 1-0.3=0.7 chance for not selling af bike.

The profit is:

$$\Pi = 0.3(1000 - w_H) + 0.7 * (0 - w_L) = 0.5 * 1000 = 300$$

It will not change the answer.

f) How will the camera affect your expected profit? And how will it affect Asger's expected utility compared to the case under c) when you did not have the camera? Calculate your profit and Asger's utility for the new scenario with the camera and compare it to your findings under c).

Now Asger wil have a 1 chance to sell a bike when e=1 and a 0 chance when e=0.

IR is:

$$1 \cdot u(e = 1, w_H) + 0 \cdot u(e = 1, w_L) \geq 0 \Leftrightarrow$$

$$1 \cdot (w_H - 90 * 1) \geq 0 \Leftrightarrow$$

$$w_H \geq 90 \tag{IR}$$

There is no IC beacuse the principal can observe Asger.

The principal has now following problem:

$$\max_{w_H} \Pi = 1 * (1000 - w_H) = 1000 - w_H \text{u.b.b}$$

$$w_H \geq 90$$

It must be true that $W_H = 90$ which i lower tha in (C) Asgers utility is lower than in C and the principal will be higher.

g) Does anybody receive information rents or pay information costs under the two situations in c) and f)? If yes, who?

In (c) the principal can't control the agents' work effort. The agent must have an incentive to work hard, higher wage. The principal has information cost, while Asger has informationreturn, which makes a higher wage for Asger than his outside utility.

In (f) there is no longer assymetric information. there is no informationcost nor return.

Question 3) 2020 sommer RE

Player 1\Player 2	Left	Right
High	6;0	5;5
Low	10;0	1;1

a) Determine all Nash equilibria in pure strategies of this simultaneous one-period game. Explain your reasoning in words.

In a Nash Equilibrium all players will play strategies that are best responses to each other. For Player 2, the best response to Player 1 playing High is Right, while the best response to Player 1 playing Low is also Right. For Player 1, the best response to Player 2 playing Left is Low, and the best response to Player 2 playing Right is High. Thus, the only combination of strategies where both players play strategies that are best responses to each other is (High; Right).

The two players now decide to make a contract. Under this contract, Player 1 is obliged to pay the amount X to Player 2 whenever Player 2 chooses the action "Left". Put differently, the contract states 4 that whenever Player 2 plays "Left", the payoff of Player 1 is reduced by X whereas the payoff of Player 2 increases by X.,

b) Write down the new payoff matrix arising from this contract.

Player 1\Player 2	Left	Right
High	6-X;0+X	5;5
Low	10-X;0+X	1;1

c) What is the minimum value of X such that (Low; Left) is a Nash equilibrium under the described contract?

c) It has to become a best response for Player 2 to play Left when Player 1 plays Low. This means that the payoff to Player 2 from playing Left has to be at least as high as the payoff from playing Right, which is 1. If Player 2 plays left, it will always be a best response of Player 1 to play Low, regardless of the value of X. Thus, we need that $X \geq 1$.

d) What condition on X must hold such that (Low; Left) is the only Nash equilibrium in this game?

From b) we know that we need that $X \geq 1$ to have (Low; Left) being a NE. The other NE is [High, Right] (from a)). We need to set X such that one player is no longer playing a best response under that NE. Playing High will always be a best response of Player 1 to Player 2 playing Right. Thus, we need to set X such that it is no longer a best response for Player 2 to play Right when Player 1 plays High. Thus we need to have that the payoff to Player 2 of playing Left when Player 1 plays High is greater than 5. We therefore need $X > 5$

e) Imagine that $X = 7$. Is there a strictly dominant strategy for any of the two players?

A strictly dominant strategy will strictly dominate all other strategies for a player regardless of what the other player does (i.e. it will give a strictly higher payoff). Player 1 will always be better off with strategy Low if Player 2 plays Left, and will always be better off with strategy High if Player 2 plays Right, regardless of the level of X, so it can only be Player 2 who might have a strictly dominant 5 strategy available. When $X = 7$, Left becomes a strictly dominant strategy for Player 2. (The only Nash equilibrium in this game will be (Low; Left), and player 2 is playing a strictly dominant strategy in it.)

Question 4

a) "Second-degree price discrimination will typically involve lower profits for a monopolist than third- degree price discrimination."

This statement is true. Second-degree price discrimination describes a situation where the monopolist knows that there are two groups of consumers, but he cannot tell them apart from each other (there is asymmetric information). Third-degree price discrimination describes a situation where there are two groups of consumers whom the monopolist can tell apart. In the latter case, the monopolist can simply charge different monopoly prices from each group, which maximizes his profits. Under second-degree price discrimination the monopolist in addition has to make sure that the packages that he offers to consumers fulfill the consumers' incentive compatibility constraints, i.e. that no consumer has an incentive to lie about her type. This will involve information cost to the monopolist and information rents to some consumers, so profits to the monopolist will tend to be lower. Alternatively, the monopolist could accept that the IC constraints will not be fulfilled and offer the same package to everybody. However, this will also tend to give lower profits than third-degree price discrimination, as now the monopolist is not doing any price discrimination anymore.

b) "The standard model of labor supply tells us that a universal basic income is distortionary. In the model a universal basic income can be viewed as an hourly tax on labor and will lead to a deadweight loss." (Note: A universal basic income is a fixed amount being paid to everybody by the government. You can neglect how the basic income is financed in your answer.)

This statement is not true. A universal basic income is a fixed amount being paid to everybody. Since it is independent of the choices of the agent, and in particular independent of the amount of labor supplied, it will not be distortionary. In the standard model of labor supply, a universal basic income will shift the budget line outward, as for any level of leisure the agent can now afford consumption that is higher by a fixed amount. This will in most cases increase both consumption and leisure, the latter meaning there will be a decrease in labor supply. However, there will not be an inward rotation of the budget line as in the case of an hourly tax on labor. If we removed the universal basic income and wanted to keep the utility of the agent constant, we would need to compensate the agent by the exact same amount of the universal basic income. This means that there is no deadweight loss.

2019 Sommer

Steel firm has a cost function of:

$$C(y) = 5y^2$$

Dmeand on steel is:

$$D(p) = \max\{600 - \frac{1}{10}p, 0\}$$

The firm has monopol

a) The quantity in equilibrium is;

First finding the inverse demand function:

$$p = 6000 - 10y$$

$$MC = 10y$$

Setting MR=MC

$$MR = 6000 - 20y$$

$$MC = MR \Leftrightarrow$$

$$10y = 6000 - 20y \Leftrightarrow y_M^* = 200$$

The price is:

$$p_M^* = 6000 - 10 * 200 = 4000$$

b) If it was perfect competition the equilibrium would be:

$$p = MC$$

$$6000 - 10y = 10y \Leftrightarrow y_{PC}^* = 300$$

$$p_{PC}^* = 6000 - 10 * 300 = 3000$$

c) CS and PS and DWL is

$$CS_M = \frac{1}{2}(6000 - 4000) \cdot 200 = 200000$$

$$PS_M = \frac{1}{2}(MC(y^*) - FC) \cdot y^* + (p^* - MC(y^*)) \cdot y^*$$

$$= \frac{1}{2}(2000 - 0) \cdot 200 + (4000 - 2000) \cdot 200 = 600000$$

$$TS = 800000$$

$$DWL = \frac{1}{2}(y_{PC}^* - y_M^*) \cdot (p_M^* - MC(y^*)) \Leftrightarrow$$

$$\frac{1}{2}(300 - 200) \cdot (4000 - 2000) = 100.00 = DWL$$

Question 3

A and B has 1 unit of goods

Edgeworth economy. Two consumers with following utility functions:

$$u_A(x_1^A, x_2^A) = \sqrt{x_1^A} + \sqrt{x_2^A}$$

$$u_B(x_1^B, x_2^B) = x_1^B + \frac{1}{2}x_2^B$$

SWF:

$$U(u_A, u_B)$$

The inital inventory is:

$$e_{x1} = e_{x1}^A + ex_1^B = 2$$

$$e_{x2} = e_{x2}^A + e_{x2}^B = 2$$

a) Assume $U(u_A, u_B) = u_A + u_B$.

Maximize SWF:

Question 1 2019 sommer RE

To agents with utillity functions:

$$u_A(x_A, g) = x_A + g$$

$$u_B(x_B, g) = x_B + 2\sqrt{g}$$

It cost 2 units of private good to buy all the public good.

The efficient condition is:

$$|MRS_A| + |MRS_B| = c'(g)$$

$$|MRS_A| = 1$$

$$|MRS_B| = \frac{1}{\sqrt{g}}$$

$$1 + \frac{1}{\sqrt{g}} = 2 \Leftrightarrow g = 1$$

b) Vil det efficiente niveau fremkomme i (Nash) ligevægten hvis agenterne individuelt træffer beslutninger om at købe (donere til) det offentlige gode?

No. With private donation will each person only focus on theur own willinges to pay (MRS) when donation and we will end up with an inefficient low level of public good.

C) Vil det efficiente nivau fremkomme i en Lindahl ligevægt?

Yes. The lindahl equilibrium implements the efficient level of the public good by introducing each person to its own price of the public good.

We have for agent i

$$u_i(x_i, g) \text{ u.b.b}$$

$$e_i = x_i + t_i g$$

$$|MRS_i| = t_i$$

We need the prices to cover the cost:

$$t_a + t_b = c'(g) \Leftrightarrow |MRS_A| + |MRS_B| = c'(g)$$

Question 2 2018 Vinter RE

We get that each unit "gartner" cost one unit of private income to finance:

$$c'(g) = 1$$

Two Agent with following utility functions:

$$u_A(G, x_A) = 2 \cdot G^{1/2} + x_A$$

$$u_B(G, x_B) = 10 * G^{1/2} + x_B$$

a) Find Lindahl-prices.

$$t_a + t_b = c'(g)$$

The lindahl price makes a public god (whis is not exluding) excluding. We can noe get rid of the freerider problem

Start by maximizing agent A's utilityproblem:

$$\max u_A = 2 \cdot G^{1/2} + x_A \text{s.t}$$

$$e_A = x_A + t_A G$$

Substituting the constraint in:

$$u_A = 2 \cdot G^{1/2} + e_A - t_A g$$

take the derivate w.r.t G:

$$\frac{\partial u_A}{\partial G} = \frac{1}{\sqrt{G}} - t_A = 0 \Leftrightarrow \frac{1}{\sqrt{G}} = t_A$$

This is equal to: $|MRS_A| = t_A$

Do the same for agent B

$$|MRS_B| = t_B \Leftrightarrow \frac{\partial u_B}{\partial G} = 5\frac{1}{\sqrt{G}} = t_B$$

$$t_A + t_B = c'(G) \Leftrightarrow \frac{1}{\sqrt{G}} + 5\frac{1}{\sqrt{G}} = 1 \Leftrightarrow G = 36$$

$$t_A = \frac{1}{\sqrt{36}} = \frac{1}{6}$$

$$t_B = 5\frac{1}{\sqrt{36}} = \frac{5}{6}$$

Question 2 2018 vinter

Grossist to 20 kr. for each firework, $MC = 20$. There are two costumers students and workers with following demand functions:

$$D_W(p) = \max\{100 - p, 0\}$$

$$D_S(p) = \max\{80 - p, 0\}$$

p is the price for each firework.

There is monopol.

Delopgave A

a) What is the price for all costumers, how many rockets do each costumer buys, how big is "dæknings-bidraget"?

Finding the aggregated demand function:

$$D(p) = 180 - 2p$$

The inverse is:

$$P(x) = 90 - \frac{1}{2}x$$

x is firework

$$MR = 90 - x$$

Finding the quantity_

$$MR = MC \Leftrightarrow 90 - x = 20 \Leftrightarrow x^* = 70$$

The price is:

$$p(x^*) = 90 - \frac{1}{2} * 70 = 55 = p^*$$

$$DB(x = 70, p = 55) = x^*(p^* - MC) = 2450$$

Maximizing the profit when we only sale to the worker:

The inverse demand function for the worker is:

$$p_w(x) = 100 - x$$

MR is

$$MR = 100 - 2x_w$$

Finding optimal quantity for the worker

$$MR = MC$$

$$100 - 2x_w = 20 \Leftrightarrow$$

$$x_w = 40$$

The price is for the worker is:

$$p_w(x_w) = 100 - 40 = 60$$

Becuase the price is 60 it would not be optimal to sell only to the worker. The students are willing to but firework to. It is optimal to sell to both costumers.

We use the old price and find:

$$p_w(x^*) = 100 - 55 = 45$$

$$p_s(x^*) = 80 - 55 = 25$$

The optimal price and quantity is:

$$(p_w, p_s, x^*, DB) = (45, 25, 55, 2450)$$

Delopgave B

He can now give sudent discount.

We can solve the max problen seperatly for the worker and students because MC is constant. In (a) we found the optimal price and quantity for the worker and we wil now start with the students:

$$p_S = 80 - x$$

$$MR_S = 80 - 2x$$

$$MC = MR \Leftrightarrow 80 - 2x = 20 \Leftrightarrow x = 30$$

$$p_S = 80 - 30 = 50$$

$$DB = DB_w + DB_S = x_w * (p_w - MC) + x_s * (p_S - MC) \Leftrightarrow$$

$$40 * (60 - 20 + 30 * (50 - 20) = 2500$$

$$(p_w, p_s, x_w, x_s, DB) = (60, 50, 40, 30, 2500)$$

Delopgave C

2. degree price discrimination

The monopolist sses the max as a principal-agent problem, where IC and IR needs to be true:

S is the price for the package and B is willinges to pay (betlaingswillighed).

$$\max \pi = S_S + S_w - MC \cdot (x_s + x_W)\text{s.t}$$

$$IR_s : B_s(x_s) - S_s \geq 0$$

$$IR_w : B_w(x_w) - S_w \geq 0$$

$$IC_S : B_s(x_s) - S_s \geq B_S(x_w) - S_w$$

$$IC_w : B_w(x_w) - S_w \geq B_w(x_s) - S_s$$

$B_s(x_s)$ and $B_S(x_w)$ is the students willingnes to pay for own package and the workers package.MC=20 kr. for one unit firework.

The willinges to pay for students and workers are:

$$B_s(x) = \int^x p_S(q)dq = \int^x (80-q)dq = 80x_s - \frac{1}{2}x_s^2$$

$$B_w(x) = \int^x p_w(q)dq = \int^x (100-q)dq = 100x_w - \frac{1}{2}x_W^2$$

The student wil not have incentive to pick the workers package, because the student's willinges to pay is lower than the worker's. IC is always true and IR is the most restrictive condition for the price of the package of the student's package.

The price of the package for the student is equal to the student's willinges to pay which gives:

$$S_S = B_S(x_S)$$

The worker will have incentive to choose the student package because the worker's willingnes to pay is higher than the student's. To secure IC_w is true we must do the worker's package more attractive by giving the worker a discount. The IR will be true and we can write the IC to:

$$B_w(x_w) - S_w = B_w(x_s) - S_s \Leftrightarrow$$

$$S_w = B_w(x_w) - B_w(x_s) + S_s = B_w(x_w) - B_w(x_s) + B_S(x_S)$$

Inserting this in max problem

$$\max \pi = B_S(x_S) + B_w(x_w) - B_w(x_s) + B_S(x_S) - 20 \cdot (x_s + x_W)$$

Max by taking the derivative w.r.t x_s

$$\frac{\partial \pi}{\partial x_s} = B'_S(x_S) - B_w(x_s) - B'_S(x_S) - 20 = 0 \Leftrightarrow$$

$$80 - x_s - (100 - x_s) + (80 - x_s) - 20 = 0 \Leftrightarrow x_s = 40$$

Do the same w.r.t x_w

$$\frac{\partial \pi}{\partial x_w} = B'_w(x_w) - 20 = 0 \Leftrightarrow$$

$$100 - x_w - 20 \Leftrightarrow x_w = 80$$

Finding the price for the student package:

$$S_S = B_S(x_S) = 80 * 40 - \frac{1}{2} * 40^2 = 2400$$

The worker package price is:

$$S_w = 100 * 80 - \frac{1}{2}80^2 - (100x_s - \frac{1}{2}x_s^2) + 2400 \Leftrightarrow$$

$$100 * 80 - \frac{1}{2}80^2 - (100 * 40 - \frac{1}{2} * 40^2) + 2400 = 4000$$

Dækningsbidraget is:

$$DB = S_w + S_s - 20 * (x_w + x_s) = 2400 + 4000 - 20 * 120 = 4000$$

Question 5 vinter 2016

Oligopol and the demand for cheese is:

$$D(p) = 120 - p$$

The two cost functions are:

$$C_A = x_A^2 * a$$

$$C_B = x_B^2 * b$$

a,b are constants greater than 0.

A) equilibrium with cournot competition

The firm compete on quantity in cournot. We start with finding the inverse demand function:

$$p(x) = 120 - x$$

$x = (x_A + x_B)$

Deriving the profit max-problem for A

$$\max \pi_A = P(x) \cdot x_A - C_A(x_A)$$

$$\pi_A = (120 - (x_A + x_B)) * x_A - x_A^2 * a$$

Max by taking the derivative w.r.t x_A and to find best response function:

$$\frac{\partial \pi_A}{\partial x_A} = 120 - 2x_A - x_B - 2x_A a = 0 \Leftrightarrow$$

$$x_A = \frac{120 - x_B}{2(1+a)}$$

Firm B and A has same cost functions which is why they must behave equal on the marked. Firm B best response function is:

$$x_B = \frac{120 - x_A}{2(1+a)}$$

Inserting x_A in x_B to find nash equilibrium (NE):

$$x_B = \frac{120 - \frac{120 - x_B}{2(1+a)}}{2(1+a)} \Leftrightarrow$$

$$x_B = \frac{240a + 120}{(4(1+a)(1+b) - 1)}$$

Firm A has the se same becuase of symmetric

$$x_A = \frac{240b + 120}{(4(1+a)(1+b) - 1)}$$

B) meaning of a and b

We see hat your own cost parameter (a-parameter for firm A and b-parameter for firm B) has a negative impact on the quantity. Bigger costs will reduce production. It is your own advantage that the other firm has high costs compared to your own firm. Higher costs weakes the competitors opportunity to compete if the the firm choose to change the production.

Question 3 2016 vinter

One package cost 20 kr. to produce and sell and no FC.

Two costumers with following demand functions:

$$D_A(p) = 140 - p$$

$$D_B(p) = 68 - p$$

Lars has monoply

a) Find price and quantity:

Deriving the aggregated inverse demand function of A:

$$P(x) = 104 + \frac{2}{x}$$

Costumer B will not buy and it is not optimal for the monoply to lower the price. So we look at when the monoply only sells to A

$$p_A(x) = 140 - x$$

Finding MR

$$MR = 140 - 2x$$

Setting MC=MR

$$140 - 2x = 20 \Leftrightarrow x = 60$$

The price is:

$$p_A(x) = 140 - 60 = 80$$

The profit is:

$$\pi = 80 \cdot 60 - 20 \cdot 60 = 3600$$

B) New demand function: $D_B(p) = 76 - p$. Answer a)

Finding the aggregated demand function:

$$D_{new}(p) = \begin{cases} 216 - 2p & 0 \leq p \leq 76 \\ 140 - p & 76 \leq p \leq 140 \end{cases}$$

The inverse is:

$$p_{new}(x) = \begin{cases} 108 - \frac{1}{2}x & 64 \leq x \leq 216 \\ 140 - x & 0 \leq x \leq 64 \end{cases}$$

MR is:

$$MR(x) = \begin{cases} 108 - x & \text{(both intering market)} \\ 140 - 2x & \text{(only A in market)} \end{cases}$$

When both in the market:

$$108 - x = 20 \Leftrightarrow x^* = 88$$

price is:

$$p^*_{new} = 108 - \frac{1}{2} * 88 = 64$$

The profit is:

$$\pi_{new} = 64 * 88 - 20 * 88 = 3872$$

We see it is optimal to sell to both costumers when we change costumer B's demand function. B's willingnes to pay is now higher which means it is optimal for the monopoly to sell to B.