Problem Set 11

Macroeconomics III

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Problem

Consider a model of monetary policy where demand and supply follows:

$$\pi_t = m_t \tag{1}$$
$$x_t = \theta_t + (\pi_t - \pi_t^e) - \epsilon_t \tag{2}$$

- π and π^e denotes actual and expected inflation. x is output.
- θ is potential output with mean of zero and variance of σ_θ^2
- ϵ is supply shock with mean of zero and variance of σ_ϵ^2

Agents form π_t^e after observing θ but not ϵ. Monetary authority sets m after observing both θ and ϵ.

The loss function covers two periods and is given by

$$L(\pi, x) = \frac{1}{2}\left[(\pi_1 - \bar{\pi})^2 + \lambda^j(x_1 - \bar{x})^2 + (\pi_2 - \bar{\pi})^2 + \lambda^j(x_2 - \bar{x})^2)\right] \tag{3}$$

There are two political parties, S and L, where $\lambda^L < \lambda^S$. Type L determines m_1 with discretion and type S determines m_2.

Finally, we assume that $\bar{\pi} = 0$ and $\bar{x} - \theta > 0$ for all θ.

a) - Inflation and output - non-cooperation

What are the equilibrium inflation and output outcomes in this economy if the political parties behave non-cooperatively?

The political parties set the monetary policy non-cooperatively, why we can solve the minimization problem for each party independently of each other.

The monetary policy rule is not credible, why they set monetary policy discretionally and minimize ex post loss function rather than ex ante (we do not minimize expected loss but instead the actual loss).

Find optimal π_t^j and the resulting x_t^j.

$$\min_{\pi_i} L(\pi_i, x_i) = \frac{1}{2}\left[(\pi_i - \bar{\pi})^2 + \lambda^j (x_i - \bar{x})^2\right] \tag{4}$$

Solve for $(i, j) \in \{(1, L), (2, S)\}$

(such that party L is in control in period 1 and party S in 2).

a) - Optimal policy - non-cooperation (1/4)

We insert the expression for x into the loss-function

$$
\begin{aligned}
L(\pi_i, x_i) &= \frac{1}{2}\left[(\pi_i - \bar{\pi})^2 + \lambda^j (x_i - \bar{x})^2\right] \\
&= \frac{1}{2}\left[(\pi_i)^2 + \lambda^j (\theta_i + \pi_i - \pi_i^e - \epsilon_i - \bar{x})^2\right]
\end{aligned}
$$

Taking the first-order condition yields:

$$
\begin{aligned}
\pi_i^D &= -\lambda^j (\theta_i + \pi_i^D - \pi_i^e - \epsilon_i - \bar{x}) \\
\pi_i^D (1 + \lambda^j) &= -\lambda^j (\theta_i - \pi_i^e - \epsilon_i - \bar{x}) \\
\pi_i^D (1 + \lambda^j) &= \lambda^j (\pi_i^e + \epsilon_i + \bar{x} - \theta_i) \\
\pi_i^D &= \frac{\lambda^j}{1 + \lambda^j} (\pi_i^e + \epsilon_i + \bar{x} - \theta_i)
\end{aligned}
$$

We then find π_i^e, which is formed conditional on θ_i

$$
\pi_i^e = \mathbb{E}[\pi_i^D | \theta_i] = \frac{\lambda^j}{1 + \lambda^j} \mathbb{E}[(\pi_i^e + \epsilon_i + \bar{x} - \theta_i) | \theta_i]
$$

a) - Optimal policy - non-cooperation (2/4)

We rewrite the expectations

$$
\begin{aligned}
\pi_i^e &= \frac{\lambda^j}{1+\lambda^j}\mathbb{E}[(\pi_i^e + \epsilon_i + \bar{x} - \theta_i)|\theta_i] \\
\pi_i^e &= \frac{\lambda^j}{1+\lambda^j}\left(\pi_i^e + \underbrace{\mathbb{E}[\epsilon_i|\theta_i]}_{=0} + \bar{x} - \theta_i\right) \\
\pi_i^e &= \frac{\lambda^j}{1+\lambda^j}\left(\pi_i^e + \bar{x} - \theta_i\right) \\
\pi_i^e\left(1 - \frac{\lambda^j}{1+\lambda^j}\right) &= \frac{\lambda^j}{1+\lambda^j}\left(\bar{x} - \theta_i\right) \\
\pi_i^e \frac{1}{1+\lambda^j} &= \frac{\lambda^j}{1+\lambda^j}\left(\bar{x} - \theta_i\right) \\
\pi_i^e &= \lambda^j\left(\bar{x} - \theta_i\right)
\end{aligned}
$$

a) - Optimal policy - non-cooperation (3/4)

We now find the optimal inflation and output.

$$
\begin{aligned}
\pi_i^D &= \frac{\lambda^j}{1+\lambda^j}(\pi_i^e + \epsilon_i + \bar{x} - \theta_i) \\
&= \frac{\lambda^j}{1+\lambda^j}\Big(\underbrace{\lambda^j(\bar{x}-\theta_i)}_{\pi_i^e} + \epsilon_i + \bar{x} - \theta_i\Big) \\
&= \frac{\lambda^j}{1+\lambda^j}\left((1+\lambda^j)(\bar{x}-\theta_i) + \epsilon_i\right) \\
&= \lambda^j(\bar{x}-\theta_i) + \frac{\lambda^j}{1+\lambda^j}\epsilon_i
\end{aligned}
$$

We can then find output

$$
\begin{aligned}
x_i^D &= \theta_i + (\pi_i - \pi_i^e) - \epsilon_i \\
&= \theta_i + \underbrace{\lambda^j(\bar{x}-\theta_i) + \frac{\lambda^j}{1+\lambda^j}\epsilon_i}_{\pi_i} - \underbrace{\lambda^j(\bar{x}-\theta_i)}_{\pi_i^e} - \epsilon_i \\
&= \theta_i - \frac{1}{1+\lambda^j}\epsilon_i
\end{aligned}
$$

a) - Optimal policy - non-cooperation (4/4)

Hence, we have the following inflation and output under discretion

$$\pi_i^D = \lambda^j(\bar{x} - \theta_i) + \frac{\lambda^j}{1+\lambda^j}\epsilon_i$$
$$x_i^D = \theta_i - \frac{1}{1+\lambda^j}\epsilon_i$$

Note: The inflation is affected by θ. $\lambda^j(\bar{x} - \theta_i)$ is the inflation bias from discretion policy. The idea is that the government will try to make stabilising monetary policy with respect to θ, which the agents know and take into account. Therefore, the government is unable to stabilise output with respect to θ and it will only affect inflation.

This does not hold for ϵ since the agents do not know ϵ when they form their expectations. Hence, the government is able to conduct output stabilisation with respect to ϵ.

b) - Optimal Choice of Central Banker - Set-Up (1/2)

Determine or characterize for which parameters λ^B party S prefers to keep the central banker in the second period. Thus, characterize the optimal choice of central banker by party L in the first period.
Related to Chapter 17.2 in reading PT15-17 on Absalon.

Period 1: L can choose an independent central banker with preferences λ^B to control monetary policy in period 1. The central banker sets monetary policy under discretion.

Period 2: S can choose to repudiate the CB and set the monetary policy under discretion or let the CB control monetary policy.

When will S keep the central banker?

Party S will choose to keep the central banker whenever

$$\mathbb{E}[L(\pi_2^B, x_2^B, \lambda^S)] \leq \mathbb{E}[L(\pi_2^S, x_2^S, \lambda^S)]$$

Where $L(\pi_2^B, x_2^B, \lambda^S) = \frac{1}{2}\left[(\pi_2^B)^2 + \lambda^S(x_2^B - \bar{x})^2\right]$.

b) - Optimal Choice of Central Banker - Set-Up (2/2)

Find the values of λ^B for which the CB is **not** repudiated by S.

$$\mathbb{E}[L(\pi_2^B, x_2^B, \lambda^S)] \leq \mathbb{E}[L(\pi_2^S, x_2^S, \lambda^S)]$$

1. Remember that π_2^B, x_2^B are functions of λ^B.
2. Take the derivative wrt. λ^B of
$$\mathbb{E}[L(\pi_2^B, x_2^B, \lambda^S)] = \frac{1}{2}\mathbb{E}\left[(\pi_2^B)^2 + \lambda^S(x_2^B - \bar{x})^2)\right]$$
3. Compute the sign of the derivative for different intervals of λ^B
4. Is the optimal value of λ^B for party S greater, smaller or equal to λ^S?
5. Compute the interval of values of λ^B for which party S will keep the CB
Hint: Might be helpful to draw the expected loss as a function of λ^B by using the sign of the derivative.

b) - Values of λ^B where CB is kept (1/3)

We consider the expected loss when keeping the central banker:

$$\mathbb{E}[L(\pi_2^B, x_2^B, \lambda^S)] = \frac{1}{2}\mathbb{E}\Big[(\pi_2^B)^2 + \lambda^S(x_2^B - \bar{x})^2\Big]$$

$$= \frac{1}{2}\mathbb{E}\left[\left(\frac{\lambda^B}{1+\lambda^B}\epsilon_2 + \lambda^B(\bar{x} - \theta_2)\right)^2 + \lambda^S\left(\theta_t - \bar{x} - \frac{1}{1+\lambda^B}\epsilon_2\right)^2\right]$$

We use that ϵ and θ are independent random variables with zero mean, why $\mathbb{E}[\epsilon \cdot \theta] = \mathbb{E}[\epsilon] \cdot \mathbb{E}[\theta] = 0$ and $\mathbb{E}[\epsilon \cdot \bar{x}] = \mathbb{E}[\theta \cdot \bar{x}] = 0$

$$= \frac{1}{2}\left[\begin{array}{l} \left(\frac{\lambda^B}{1+\lambda^B}\right)^2 \mathbb{E}[\epsilon_2^2] + (\lambda^B)^2(\bar{x}^2 + \mathbb{E}[\theta^2]) \\ +\lambda^S\left[(\bar{x}^2 + \mathbb{E}[\theta^2]) + \left(\frac{1}{1+\lambda^B}\right)^2 \mathbb{E}[\epsilon_2^2]\right] \end{array}\right]$$

Since ϵ and θ have zero means we know that $\mathbb{E}[\epsilon^2] = \sigma_\epsilon^2$ and $\mathbb{E}[\theta^2] = \sigma_\theta^2$.

$$= \frac{1}{2}\left[\left(\frac{\lambda^B}{1+\lambda^B}\right)^2 \sigma_\epsilon^2 + (\lambda^B)^2(\bar{x}^2 + \sigma_\theta^2) + \lambda^S\left((\bar{x}^2 + \sigma_\theta^2) + \left(\frac{1}{1+\lambda^B}\right)^2 \sigma_\epsilon^2\right)\right]$$

b) - Values of λ^B where CB is kept (2/3)

We now have an expression for the expected loss function

$$\frac{1}{2}\left(\left(\frac{\lambda^B}{1+\lambda^B}\right)^2\sigma_\epsilon^2+(\lambda^B)^2(\bar{x}^2+\sigma_\theta^2)+\lambda^S\left[(\bar{x}^2+\sigma_\theta^2)+\left(\frac{1}{1+\lambda^B}\right)^2\sigma_\epsilon^2\right]\right)$$

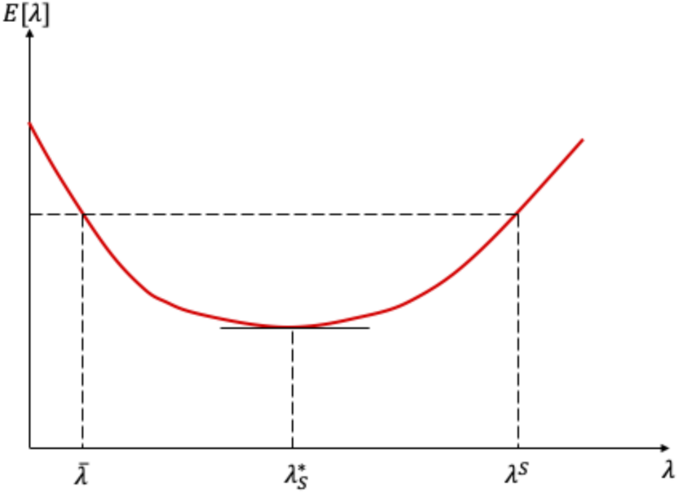
We find the derivative wrt. λ^B to find the optimal λ^B

$$\begin{aligned}\frac{\partial\mathbb{E}[L]}{\partial\lambda^B}&=\left(\frac{\lambda^B}{1+\lambda^B}\right)\left(\frac{(1+\lambda^B)-\lambda^B}{(1+\lambda^B)^2}\right)\sigma_\epsilon^2+\lambda^B(\bar{x}^2+\sigma_\theta^2)-\left(\frac{\lambda^S\sigma_\epsilon^2}{(1+\lambda^B)^3}\right)\\&=\left(\frac{\lambda^B-\lambda^S}{(1+\lambda^B)^3}\right)\sigma_\epsilon^2+\lambda^B(\bar{x}^2+\sigma_\theta^2)\end{aligned}$$

We then continue by investigating the sign of the derivative and define $\hat{\lambda}^S$ as the optimal value of λ^B for party with preferences λ^S.

$$\frac{\partial\mathbb{E}[L]}{\partial\lambda^B}=\begin{cases}<0 \text{ for } \lambda^B<\hat{\lambda}^S\\=0 \text{ for } \lambda^B=\hat{\lambda}^S<\lambda^S\\>0 \text{ for } \lambda^B>\hat{\lambda}^S\end{cases}$$

b) - Values of λ^B where CB is kept (3/3)



Party S will keep the central banker if the central banker has preferences,

$$\lambda^B \in [\bar{\lambda}^S, \lambda^S]$$

b) - Optimal λ^B for party L (1/2)

Party S will keep the central banker if the central banker has preferences,

$$\lambda^B \in [\bar{\lambda}^S, \lambda^S]$$

Where $\bar{\lambda}^S$ solves

$$\mathbb{E}[L(\pi_2^B, x_2^B, \lambda^S)] = \mathbb{E}[L(\pi_2^S, x_2^S, \lambda^S)]$$

We know that $\lambda^L < \lambda^S$, which implies that $\hat{\lambda}^L < \hat{\lambda}^S$.

Scenario 1: If the optimal output volatility preference of party L is within that interval, party L will just the central banker such that:

$$\lambda^B = \hat{\lambda}^L \in [\bar{\lambda}^S, \lambda^S]$$

By doing so, the monetary policy will be optimally set for party L in both periods.

b) - Optimal λ^B for party L (2/2)

Scenario 2: If the optimal preferences are outside the acceptance region, $\hat{\lambda}^L < \bar{\lambda}^S$, then party L has two options.

1. Set $\lambda^B = \hat{\lambda}^L$ such that monetary policy is optimal in period 1. Party S will then repudiate the central banker. The expected loss will be,

$$\mathbb{E}\Big[L(\hat{\pi}_1^L, \hat{x}_1^L, \lambda^L) + L(\pi_2^S, x_2^S, \lambda^L)\Big]$$

   Where $\hat{\pi}_1^L$ and $\hat{x}_1^L$ denotes the inflation and output obtained by the optimal central banker.

2. Set $\lambda^B = \bar{\lambda}^S$ such that party S keeps the central banker. This can be seen as cooperation or a compromise. The expected loss will then be

$$\mathbb{E}\Big[L(\pi_1^{coop}, x_1^{coop}, \lambda^L) + L(\pi_2^{coop}, x_2^{coop}, \lambda^L)\Big]$$

   Where π^{coop} and x^{coop} denotes the inflation and output obtained by the central banker with preferences $\lambda^B = \bar{\lambda}^S$.

c) - Currency Peg

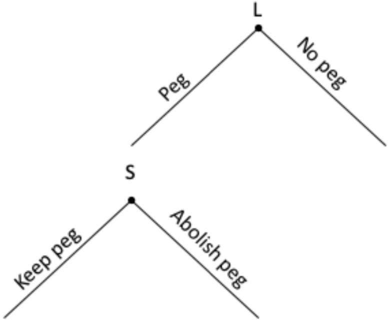
Determine under what circumstances party S prefers to keep the peg in the second period. Determine when it is optimal to party L to fix the exchange rate in the first period.

Currency peg implies,

$$\pi^P = \pi^* = 0$$

Party L can establish the currency peg in the first period and party S can either keep or abolish it in the second period.

The currency peg is not credible in the first period but becomes credible if it is kept in the second period - that is $\pi_2^e = 0$ if it is kept.



c) - When Will S Keep the Peg? (1/4)

If party S keeps the peg, they will set inflation to zero such that,

$$\pi_2^P = \pi_2^e = 0$$
$$x_2^P = \theta_2 - \epsilon_2$$

The expected loss will then be

$$\begin{aligned}
\mathbb{E}\Big[L(\pi_2^P, x_2^P, \lambda^S)\Big] &= \mathbb{E}\left[\frac{1}{2}\lambda^S(\theta_2 - \bar{x} - \epsilon_2)^2\right] \\
&= \frac{1}{2}\lambda^S \mathbb{E}\left[(\theta_2 - \bar{x} - \epsilon_2)^2\right] \\
&= \frac{1}{2}\lambda^S(\sigma_\theta^2 + \bar{x}^2 + \sigma_\epsilon^2)
\end{aligned}$$

Hence, we have the expected loss if they keep the peg.

c) - When Will S Keep the Peg? (2/4)

If party S abandon the peg, they will set monetary policy under discretion and expectations will be formed accordingly such that,

$$\pi_2^D = \lambda^S(\bar{x}-\theta_2) + \frac{\lambda^S}{1+\lambda^S}\epsilon_2, \quad \pi_2^e \quad = \lambda^S(\bar{x}-\theta_2), \quad x_2^D = \theta_2 - \frac{1}{1+\lambda^S}\epsilon_2$$

The expected loss will then be:

$$\begin{aligned}
&\mathbb{E}\Big[L(\pi_2^D, x_2^D, \lambda^S)\Big] = \frac{1}{2}\mathbb{E}\Big[(\pi_2^D)^2 + \lambda^S(x_2^D - \bar{x})^2\Big] \\
&= \frac{1}{2}\mathbb{E}\left[\left(\frac{\lambda^S}{1+\lambda^S}\epsilon_2 + \lambda^B(\bar{x}-\theta_2)\right)^2 + \lambda^S\left(\theta_t - \bar{x} - \frac{1}{1+\lambda^S}\epsilon_2\right)^2\right] \\
&= \frac{1}{2}\left(\left(\frac{\lambda^S}{1+\lambda^S}\right)^2\sigma_\epsilon^2 + (\lambda^S)^2(\bar{x}^2+\sigma_\theta^2) + \lambda^S\left[(\bar{x}^2+\sigma_\theta^2) + \left(\frac{1}{1+\lambda^S}\right)^2\sigma_\epsilon^2\right]\right) \\
&= \frac{1}{2}\left(\frac{(\lambda^S)^2+\lambda^S}{(1+\lambda^S)^2}\sigma_\epsilon^2 + \lambda^S(1+\lambda^S)(\bar{x}_2^2+\sigma_\theta^2)\right) \\
&= \frac{1}{2}\left(\frac{\lambda^S}{1+\lambda^S}\sigma_\epsilon^2 + \lambda^S(1+\lambda^S)(\bar{x}_2^2+\sigma_\theta^2)\right)
\end{aligned}$$

c) - When Will S Keep the Peg? (3/4)

They will keep the peg as long as the expected loss from the peg is smaller than the expected loss from discretion.

$$
\begin{aligned}
\mathbb{E}\Big[L(\pi_2^P, x_2^P, \lambda^S)\Big] &\leq \mathbb{E}\Big[L(\pi_2^D, x_2^D, \lambda^S)\Big] \\
\frac{1}{2}\lambda^S(\sigma_\theta^2 + \bar{x}^2 + \sigma_\epsilon^2) &\leq \frac{1}{2}\left(\frac{\lambda^S}{1+\lambda^S}\sigma_\epsilon^2 + \lambda^S(1+\lambda^S)(\bar{x}_2^2 + \sigma_\theta^2)\right) \\
\lambda^S(\sigma_\theta^2 + \bar{x}^2 + \sigma_\epsilon^2) &\leq \lambda^S\left(\frac{1}{1+\lambda^S}\sigma_\epsilon^2 + (1+\lambda^S)(\bar{x}_2^2 + \sigma_\theta^2)\right) \\
\sigma_\theta^2 + \bar{x}^2 + \sigma_\epsilon^2 &\leq \frac{1}{1+\lambda^S}\sigma_\epsilon^2 + (1+\lambda^S)(\bar{x}_2^2 + \sigma_\theta^2) \\
\sigma_\epsilon^2 &\leq \frac{1}{1+\lambda^S}\sigma_\epsilon^2 + \lambda^S(\bar{x}_2^2 + \sigma_\theta^2) \\
\left(1 - \frac{1}{1+\lambda^S}\right)\sigma_\epsilon^2 &\leq \lambda^S(\bar{x}_2^2 + \sigma_\theta^2) \\
\frac{\lambda^S}{1+\lambda^S}\sigma_\epsilon^2 &\leq \lambda^S(\bar{x}_2^2 + \sigma_\theta^2) \\
\sigma_\epsilon^2 &\leq (1+\lambda^S)(\bar{x}_2^2 + \sigma_\theta^2)
\end{aligned}
$$

c) - When Will S Keep the Peg? (4/4)

We know that party S will keep the currency peg if the following condition holds:

$$\sigma_\epsilon^2 \leq (1 + \lambda^S)(\bar{x}_2^2 + \sigma_\theta^2) \tag{5}$$

The currency peg removes the opportunity of making stabilisation policy in response to the supply shock ϵ. On the other, hand the currency peg removes the inflation bias that is present for discretionary policy.

- σ_ϵ affects how much the supply shock affects the economy. If σ_ϵ is large, it will be costly not to be able to conduct stabilising monetary policy.
- $(1 + \lambda^S)(\bar{x}_2^2 + \sigma_\theta^2)$ affects how large the inflation bias will be. The inflation bias increases the expected loss, why this can be seen as the cost of making discretionary policy.

c) - When Will L Prefer the Peg?

Adopting the currency peg in the first period is costly since inflation expectations will be based on discretionary monetary policy. Therefore, party L will never adopt the currency peg if (5) is not fulfilled.

If (5) is not met, party L will perform discretionary policy.

If (5) is fulfilled, there are two scenarios:

1. L establishes currency peg and S keeps it in place. In this case, L incur a large loss in period 1 due to the lack of credibility,

$$\begin{aligned} \pi_1 &= 0 \\ x_1 &= \theta_1 - \lambda^L(\bar{x} - \theta_1) - \epsilon_1 \end{aligned}$$

2. Both L and S perform discretionary monetary policy as in question a.

L will choose the strategy with the lowest expected loss.