



Faculty of Social Sciences



Corporate Finance Theory

Lecture 4

The Leverage Ratchet Effect (1) Admati et al. (2018)

Nick Vikander
Department of Economics



Intended outcomes for the day:

1. To understand why Admati et al. (2018) assume that shareholders, when buying back debt, must pay debtholders the post-buyback price (of debt)
2. To mathematically derive how shareholders will suffer from buying back debt, even if the debt reductions increase total firm value.
3. To show how shareholder losses from buying back debt depend on factors such as taxes, default costs, and the ability to negotiate with debtholders



Main Message of the Paper

Inefficiency of capital structure dynamics

Firms with debt may resist buying back debt (recapitalization), even if it is efficient. They may instead issue more debt, even if it is inefficient

Focus on private efficiency (firm value), not social efficiency (social welfare)

Modelled as an agency conflict: managers act on behalf of shareholders, pass costs on to creditors

In this course, we will concentrate on Admati et al. (2018)'s static model:
Section II.



Introduction: Notation

$\mathbf{x}$: cash flow, random variable

$\tilde{\mathbf{x}}$: realized cash flow

$\mathbf{D}$: face value of debt

$\mathbf{t}(\tilde{\mathbf{x}}, \mathbf{D})$: tax on shareholder profits, decreasing in D

$\mathbf{n}(\tilde{\mathbf{x}}, \mathbf{D})$: net default (bankruptcy) costs

interest
tax shield

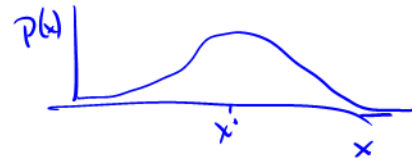
solvency

	If $\tilde{x} < D$	If $\tilde{x} \geq D$
Payoff to Shareholders	0	$\tilde{x} - t(\tilde{x}, D) - D$
Payoff to Debtholders	$\tilde{x} - n(\tilde{x}, D)$	D

default

↑ expected cash flow going to creditors $\equiv$ "market value of debt"

① Let cash flow x lie between 0 and ∞ . Density function $f(x)$



$$\text{Total value of debt} = V^D(D) = \int_D^\infty \underbrace{D}_{\text{solvency}} dF(x) + \int_0^D \underbrace{(x - n(x, D))}_{\text{default}} dF(x), \quad (1)$$

$$\text{Value of equity} = V^E(D) = \int_D^\infty \underbrace{(x - t(x, D) - D)}_{\text{solvency}} dF(x). \quad (2)$$

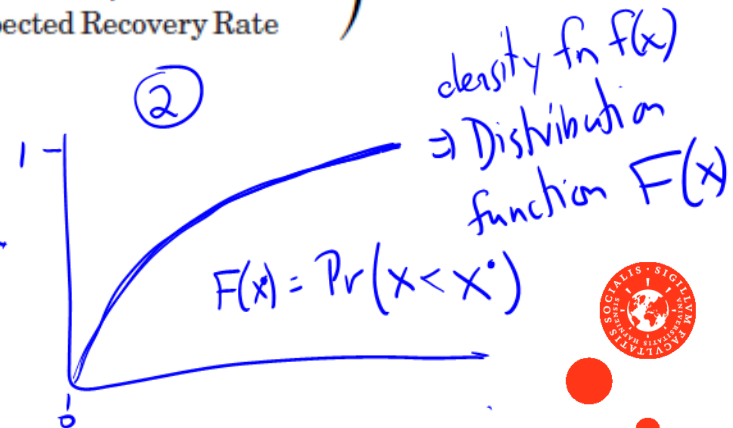
↑ expected cash flow going to shareholders

$$q(D) = \frac{V^D(D)}{D} = 1 - \underbrace{F(D)}_{\text{Probability of Default}} \left(1 - \underbrace{E \left[\frac{\tilde{x} - n(\tilde{x}, D)}{D} \middle| \tilde{x} < D \right]}_{\text{Expected Recovery Rate}} \right), \quad (3)$$

↑ Price of debt.

③ $F(x) = \int_0^x f(t) dt$
 $dF(x) = f(x) dx$

②



Shareholder gain from buying back debt

PROPOSITION 1 (Shareholder Resistance to Leverage Reduction): *Equity holders are strictly worse off issuing securities to recapitalize the firm and reduce its outstanding debt. Losses to equity holders arise from the loss of their default option, the reduction in dilution of existing debt, and higher taxes. The loss to equity holders increases with debt tax shields and recovery rates.*

Gain to shareholders from reducing their debt | To be concrete, reduce debt from D to $D-d$.

$$G(D, D-d) \equiv \underbrace{V^E(D-d)}_{\text{Value of equity after reducing debt}} - \underbrace{V^E(D)}_{\text{Value of equity before reducing debt}} - \underbrace{q(D-d)d}_{\text{Amount paid to creditors to reduce debt.}}$$

Value of equity after reducing debt

Value of equity before reducing debt.

Amount paid to creditors to reduce debt.

Assumption: debt trades at post-buy back prices.

$$q(D-d) > q(D).$$

This is expensive!

Question: why post-buyback prices?

The price of debt is $q(D - d) > q(D)$. That is, **they assume debt trades at post-buyback prices.**

To understand why, consider the following thought experiment

-You are one of 100 debtholders. Each one of you holds a bond of face value 1. Thus, the total face value of debt is equal to 100

-Firm cash flow is equal to 70, for sure. Thus, you expect to be repaid 0.7.

-Now suppose that shareholders approach 60 debtholders, and offer to buy back each of their debt (i.e. to repurchase their bonds) at price q .

-How high does q have to be in order for the 60 debtholders all to accept the shareholders' offer?

That is: what is the price "q" at which the shareholders are able to reduce (i.e. buy back) their debt?

Go to socrative.com, room 897458, and vote

A. $q = 0.3$

B. $q = 0.5$

C. $q = 0.7$

D. $q = 1.0$



Game between debtholders: accept/reject. What is the Nash equilibrium?

Candidate equilibrium: offer price q , each of the 60 debtholders accepts

For any one of these debtholders:

Equilibrium payoff = q , i.e. the amount they receive

Deviation payoff = *If I do not sell, while everyone else does, the firm's debt is reduce to ≈ 40 . $\rightarrow$ I will get 1.*

Thus, debtholders will only accept if the firm offers $q = 1$

The firm wants to buy back debt with market value $60 * \underline{q}(100) = 60 * \underline{0.7} = \underline{42}$.

But to do so, it has to pay $60 * q(\underline{40}) = 60 * \underline{1} = \underline{60}$.

Implicitly, in this thought experiment:

- Many dispersed creditors
- Shareholders offers a price q that makes all 60 debtholders buy
- Best response, given the decisions of other debtholders
- Each debtholder take others' decisions as given

Perhaps less reasonable in other settings:

- Suppose instead of 60 debtholders each with debt 1, there are 2 debtholders each with debt 30
- For any q , Nash equilibrium of the debtholder game is just as before
- In practice shareholders may be able to negotiate with debtholders in a situation like this, get them to accept a lower price



Shareholder gain from buying back debt

Now we show that the shareholder gain, from reducing debt from D to $D - b$, is just as described in the proof of Proposition 1

$$\underline{G(D, D - d)} \equiv \underline{V^E(D - d)} - \underline{V^E(D)} - \underline{q(D - d)d} < \bigcirc$$



Shareholder gain from buying back debt

$$G(D, D-d) \equiv V^E(D-d) - V^E(D) - q(D-d)d$$

write this out term by term

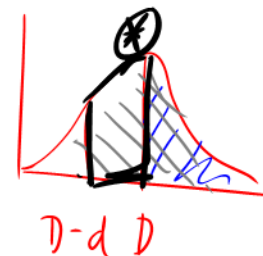
$$\int_{D-d}^{\infty} \underbrace{x - t(x, D-d)}_{\text{red squiggle}} - \underbrace{(D-d)}_{\text{green squiggle}} dF(x)$$

$$- \int_D^{\infty} \underbrace{x - t(x, D)}_{\text{red squiggle}} - \underbrace{D}_{\text{red squiggle}} dF(x) - q(D-d)d$$

$$= \int_{D-d}^D \underbrace{x - D}_{\text{red squiggle}} dF(x) - \int_{D-d}^D \underbrace{t(x, D-d)}_{\text{blue squiggle}} dF(x) + \int_D^{\infty} \underbrace{t(x, D)}_{\text{blue squiggle}} dF(x)$$

$$+ d \int_{D-d}^{\infty} dF(x) - d \cdot q(D-d)$$

$$d [1 - F(D-d) - q(D-d)]$$



Shareholder gain from buying back debt

$$G(D, D - d) \equiv V^E(D - d) - V^E(D) - q(D - d)d$$



Shareholder gain from buying back debt

$$G(D, D - d) \equiv V^E(D - d) - V^E(D) - q(D - d)d$$



$$\begin{aligned}
G(D, D-d) &\equiv V^E(D-d) - V^E(D) - q(D-d)d \\
&= \int_{D-d}^D (x-D) dF(x) < 0 \\
&\quad + d \times (1 - F(D-d) - q(D-d)) < 0 \\
&\quad + \int_D^\infty t(x, D) dF(x) - \int_{D-d}^\infty t(x, D-d) dF(x). < 0
\end{aligned}$$

Above expression is strictly negative: **shareholders resist debt reductions**

Even in situations where reducing debt increases firm value (i.e. is efficient).

- E.g. situations where $t = 0$ and $n > 0$
- No tax benefit from debt in case of solvency
- Default costs in case of bankruptcy



$$\begin{aligned}
 G(D, D-d) &\equiv V^E(D-d) - V^E(D) - q(D-d)d \\
 &= \int_{D-d}^D \underbrace{(x-D)}_{\text{red underline}} dF(x) \\
 &\quad + d \times (1 - F(D-d) - q(D-d)) \\
 &\quad + \int_D^\infty t(x, D) dF(x) - \int_{D-d}^\infty t(x, D-d) dF(x).
 \end{aligned}$$

Loss of default option effect

Shareholders have to pay debtholders in full when the realized cash flow x is between $D-d$ and D .

$$\begin{aligned}
 G(D, D-d) &\equiv V^E(D-d) - V^E(D) - q(D-d)d \\
 &= \int_{D-d}^D (x-D) dF(x) \\
 &\quad + d \times (1 - F(D-d) - q(D-d)) < 0. \\
 &\quad + \int_D^\infty t(x, D) dF(x) - \int_{D-d}^\infty t(x, D-d) dF(x).
 \end{aligned}$$

take definition from slide 5
Substitute.

Reverse dilution effect

When buying back debt, shareholders must pay debtholders for the amount debtholders expect to recover under default, if they were to hold onto their debt.

$$-d \times \underbrace{F(D-d)}_{\text{Probability of Default}} \times \underbrace{E\left[\frac{\tilde{x} - n(\tilde{x}, D-d)}{D-d} \mid \tilde{x} < D-d\right]}_{\text{Expected Recovery Rate}} \leq 0. \quad (6)$$

$$G(D, D-d) \equiv V^E(D-d) - V^E(D) - q(D-d)d$$

$$= \int_{D-d}^D (x-D) dF(x) + d \times (1 - F(D-d) - q(D-d)) + \int_D^{\infty} t(x, D) dF(x) - \int_{D-d}^{\infty} t(x, D-d) dF(x).$$

Tax effect

by assumption
that $t(x, D)$ decreases
in D .

Shareholders are now more likely to pay (higher) taxes: debt reduction increases the probability of solvency, and can reduce the interest tax shield.

Note: lower taxes (say $t = 0$) will **decrease** the shareholders' incentive to reduce debt.

$$t(x, D-d) > t(x, D).$$

Question: Impact of default costs on shareholder incentive to buy back debt

$$\begin{aligned}
 G(D, D-d) &\equiv V^E(D-d) - V^E(D) - q(D-d)d \\
 &= \int_{D-d}^D (x-D) dF(x) \\
 &\quad + d \times (1 - F(D-d) - q(D-d)) \\
 &\quad + \int_D^\infty t(x, D) dF(x) - \int_{D-d}^\infty t(x, D-d) dF(x).
 \end{aligned}$$

Question

Will lower default costs (i.e. a reduction in $n(x, D)$) **increase** or **decrease** the shareholders' incentives to reduce debt? Will it do via the "loss of default option effect", or the "reverse dilution effect" effect?

Discuss with your neighbor. Then go to socrative.com, Room 897458, and vote for the best answer

- A.** Increase via loss of default option effect; **B.** Increase via reverse dilution effect
C. Decrease via loss of default option effect; **D.** Decrease via reverse dilution effect

Will lower default costs (i.e. a reduction in $n(x,D)$) **increase** or **decrease** the shareholders' incentives to reduce debt? Will it do by via the "loss of default option effect", the "reverse dilution effect" effect, or the "tax effect"?

We look at a situation where default costs, n , go down.

Then the price of debt, q , to go up. This term was in the "reverse dilution effect", underlined in green.

A high price of debt makes it more costly for the shareholders to buy back debt.

Hence, we expect, that when n goes down (low default costs), shareholders find reducing debt even more unattractive.

Correct answer is D.



Debtholder gains and shareholder losses

Now assume there are no taxes or default costs: $t = n = 0$

We can show $-G(D, D-d) = D [q(D-d) - q(D)]$

Interpretation: Shareholder losses = Creditor gains

Buying back debt zero NPV transaction

-Left-hand side are shareholder losses from debt reduction

-Right-hand side are debtholder gains

-Notice that $q(D - d) > q(D)$. Lower debt means that default is less likely, i.e. debtholders are more likely to be repaid in full

$$G(D, D-d) \equiv V^E(D-d) - V^E(D) - q(D-d)d$$

Now assume there are no taxes or default costs: $t = n = 0$

We can show $\cancel{-G(D, D-d)} = \cancel{D [q(D-d) - q(D)]}$

$$\begin{aligned}
 \underline{V^E(D)} - \underline{V^E(D-d)} &= \underline{(D-d)} \underline{q(D-d)} - \underline{D} \underline{q(D)} \\
 &= \cancel{(D-d)} \frac{V^D(D-d)}{\cancel{D-d}} - \cancel{D} \frac{V^D(D)}{\cancel{D}} \\
 &= V^D(D-d) - V^D(D)
 \end{aligned}$$

$$\Rightarrow V^E(D) + V^D(D) = V^E(D-d) + V^D(D-d).$$

$$\begin{aligned}
 \int_0^\infty x \cancel{dF(x)} &= \int_0^\infty x dF(x) \\
 \text{LHS} &= \text{RHS}
 \end{aligned}$$

Now assume there are no taxes or default costs: $t = n = 0$

We can show $-G(D, D-d) = D [q(D-d) - q(D)]$



Now assume there are no taxes or default costs: $t = n = 0$

We can show $-G(D, D-d) = \underbrace{D [q(D-d) - q(D)]}$

Two groups of creditors.

First, those who sell their bonds. How much do they get? Their gain: $d [q(D-d) - q(D)]$

Debtholders who sell enjoy a total gain of $d [q(D-d) - q(D)]$

Debtholders who do not sell enjoy a total gain of $(D - d) [q(D-d) - q(D)]$

Collective bargaining: negotiating with debtholders

Continue to assume $t = n = 0$

We just showed that: $-G(D, D-d) = D [q(D-d) - q(D)]$

Now suppose we allow for the possibility of collective bargaining:

- Shareholders can negotiate with debtholders,
- Strike a deal to buy back debt d at price q , where $q(D) < q < q(D-d)$.

Then loss shareholders suffer from buying back debt will **decrease**

$-G(D, D-d) =$

$$d[q - q(D)] + (D-d) [q(D-d) - q(D)] <$$

$$D [q(D-d) - q(D)]$$

but *they will still suffer a loss*



Collective bargaining: negotiating with debtholders

Suppose that we allow for the possibility of collective bargaining:

- Shareholders can negotiate with creditors,
- Strike a deal to buy back debt d at price q , where $q(D) < q < q(D-d)$.

Shareholder loss:

$$-G(D, D - d) = d[q - q(D)] + (D-d) [q(D-d) - q(D)]$$

Only in the one special case (i.e. specific parameter values for d and q) will shareholders not suffer a loss, and thus be willing to buy back debt.



Intended outcomes revisited

1. To understand why Admati et al. (2018) assume that shareholders, when buying back debt, must pay debtholders the post-buyback price (of debt)
 - Best response for each creditor to hold out, unless offered the post buy back price. Market based approach.
 - More reasonable with small and dispersed creditors (no negotiation)
1. To mathematically derive how shareholders will suffer from buying back debt, even if the debt reductions increase total firm value.
 - Loss of default option effect, reverse dilution effect, tax effect.
 - Efficiency gains captured by debtholders
2. To show how shareholder losses from buying back debt depend on factors such as taxes, default costs, and the ability to negotiate with debtholders
 - Shareholder losses are increasing in taxes, decreasing in default costs, decreasing in "ability" to negotiate with debtholders



For next time

We will continue looking at Admati et al. (2018), and continue to focus on Section II.

When reading Section II, pay particular attention to

- Proposition 4, and the discussion surrounding this proposition (in subsection D).
- The discussion of how debt seniority (i.e. junior vs senior debt) affects shareholder incentives to buy back this debt
- Complete the activity posted in the Absalon module for Lecture 5, comparing the leverage ratchet effect to trade off theory. Be ready to share your thoughts next time!

