



Faculty of Social Sciences



Corporate Finance Theory

Lecture 2 Financing under moral hazard (ii)

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Announcement – Aarhus Symposium

<https://vimeo.com/863232973?share=copy>



Intended outcomes for the day:

1. To describe how the potential for credit rationing can result in borrowers holding “inside equity”, with lenders issuing “outside debt”.
2. To derive how diversification (cross-pledging the returns of different projects) can affect credit rationing, and make the connection with asset substitution.
3. To extend the model of credit rationing in order to incorporate liquidity management, and show how its results relate to free-cash flow theory, debt overhang, and dilution.



Last lecture

Market imperfections can affect firm financing decisions

Taxes, Moral hazard, Adverse selection

We worked with a model of credit rationing, where in particular:

- Lenders are only confident the borrower will behave if the latter has a sufficient stake in project success (i.e. 'skin in the game')
- Positive NPV projects may not get funded (credit rationing)
- Relatedly, a borrower with initial debt may be unable to borrow from new lenders to finance a new positive-NPV project (debt overhang)
- Lenders may suffer from a borrower taking on additional debt from new lenders, even if the new debt is junior to their own (dilution). See video.

Return: $R, 0$
 P_H, P_L
 Private benefit
 $B > 0$
 $\underline{R}_b + \underline{R}_L = \underline{R}$



Question from last time

Consider the model of credit rationing from Lecture 1 (slides 20 – 29) but with one difference: project failure still yields strictly cash flow. That is, failure yields R' , with $0 < R' < R$.

A contract now must specify the cash flow the borrower and lender receive in case of success, R_b and R_l respectively; and the cash they receive in case of failure, R_b' and R_l' .

- i) For a given contract, write down the incentive compatibility constraint for the borrower to engage in good behavior.
- ii) If lenders are willing to break even on average, under what condition will the project get financed?

Hint: follow the steps seen in the slides for Lecture 1 .
Be ready to share your thoughts!

Note: you can assume
 $p_L R + (1 - p_L) R' + B < I$

(R_b, R_l)
 $R_b + R_l = R$
 Failure
 $\rightarrow$ both get nothing.



i) For a given contract, write down the incentive compatibility constraint for the borrower to engage in good behavior.

ii) If lenders are willing to break even on average, under what condition will the project get financed?

must be higher

- A. (i) $IC_A: (p_H - p_L)(R_b - R'_b) \geq B$ (ii) $p_H(R - R_b) + (1 - p_H)(R' - R'_b) \geq I - A$ and IC_A
- B. (i) $IC_B: p_H R_b - p_L R'_b \geq B$ (ii) $p_H(R - R_b) + (1 - p_H)(R' - R'_b) \geq I - A$ and IC_B
- C. (i) $IC_C: (p_H - p_L)(R_b - R'_b) \geq B$ (ii) $p_H R + (1 - p_L)(R' - R'_b) \geq I - A$ and IC_C
- D. (i) $IC_D: p_H R_b - p_L R'_b \geq B$ (ii) $p_H R + (1 - p_L)(R' - R'_b) \geq I - A$ and IC_D

Discuss in pairs. Then go to Socrative.com, room 897458, and vote for what you think is the best answer.



Borrower Incentive constraint:

new

$$\underbrace{p_H R_B + (1-p_H) R_B'}_{\text{diligent}} \geq \underbrace{p_L R_B + (1-p_L) R_B'}_{\text{not}} + B$$

That is:

$$(p_H - p_L)(R_B - R_B') \geq B \quad (A \text{ and } C).$$

Otherwise, project has negative NPV, will never be financed:

Just like in last lecture

Lender break-even condition:

$$\underbrace{p_H R_L + (1-p_H) R_L'}_{\text{get out on average}} = \underbrace{I - A}_{\text{in}}$$

That is:

using

$$\left. \begin{array}{l} R_B + R_L = R \\ R_B' + R_L' = R' \end{array} \right\} p_H (R - R_B) + (1-p_H)(R' - R_B') \geq I - A$$

Answer A

Optimal contract

Invest: $I - A$. $R_L = R - R_B$
 who get what $R_L' = R' - R_B$.

Consider contract (R_B, R_B') ; with $0 \leq R_B \leq R$ and $0 \leq R_B' \leq R'$

Conditional on the project getting funded, and the borrower behaving, the borrower earns:

$$U_B = p_H R_B + (1 - p_H) R_B' - A$$

If the contract allows lenders to break even on average:

$$p_H(R - R_B) + (1 - p_H)(R' - R_B') = I - A \quad (\neq)$$

Then the borrower's payoff reduces to

$$U_B = p_H R + (1 - p_H) R' - I$$

} Project NPV.

There are many contracts (R_B, R_B') that allow the lender to break even on average. Which one(s) is/are optimal for the borrower?

Optimal for the borrower to offer the contract that makes their incentive constraint easiest to satisfy:

$$(p_H - p_L)(R_b - R'_b) \geq B$$

large *small*

If possible: would like to set R_b high and R'_b low. Suppose the lender's break-even constraint binds

$$p_H(R - R_b) + (1 - p_H)(R' - R'_b) = I - A$$

and $R'_b > 0$. Now consider decreasing R'_b by $\varepsilon > 0$ and increasing R_b by:

$$\left(\frac{1 - p_H}{p_H} \right) \varepsilon$$

unchanged

LHS is just as before

The borrower's incentive constraint is relaxed. The lender's break-even constraint still binds.

Conclusion: set $R_b' = 0$. Given this contract (and conditional on the project getting funded) ...

In case of failure:

- Borrower receives nothing
- Lenders receive entire cash flow $R_L' = R'$

In case of success:

- Borrower receives $R_b > 0$
- Lenders receive a share of the cash flow $R_L = R - R_b$

Interpretation: INSIDE EQUITY AND OUTSIDE DEBT

Giving lenders a debt claim ensures that the borrower earns nothing in case of project failure.

Gives the borrower the strongest possible incentive to behave

(since $R_b' = 0$),
 Lender get $R_L' = R'$
NO MATTER WHAT
 $R_L > R'$; $R_L' = \text{safe debt}$
 $R_L - R_L' = \text{risky debt}$



Lecture 1: $R_B \rightarrow \text{Borrower}$; $R_B + R_L = R$.
 $R_L \rightarrow \text{Lender}$

Interpret as debt: R_L , borrowed from lender.
 Repaid only in case of success.

Interpret as equity: share: $\rightarrow X R = R_L$.
 share: $X = \frac{R_L}{R}$ in the firm

Lecture 1: debt, equity R interpretations are equivalent.

Lecture 2: no longer the case.

safe $\downarrow$ debt: R_L'
contract.

Diversification

Can diversification help alleviate credit rationing?

Consider a borrower with two independent, identical projects. Each requires investment I , and the borrower has assets $2A$.

Otherwise, just as before: *→ Lecture 1*

- Probabilities p_H and p_L
- Cash flow R for success and 0 for failure
- Private benefit B from borrower misbehavior
- Projects only have positive NPV under good behavior:

$$p_H R > I > p_L R + B$$

Project (separate) financing: investors for a project only have a claim on the returns from that particular project.

Just as Lecture 1

pledgeable income per project.

$$p_H \left(R - \frac{B}{\Delta p} \right) \geq \underline{I} - \underline{A}$$



Cross-Pledging (joint financing): investors for a particular project have claim on the returns from the other project.

Notation:

R2 : Borrower reward when 2 projects succeed.
 R1 : " " " 1 project succeeds.
 R0 : " " " 0 projects succeed.

Borrower's expected payoff (gross of costs) :

$$\underbrace{p_H^2}_{\uparrow \text{ 2 successes }} R_2 + \underbrace{2p_H}_{\uparrow \text{ 1 success }} (1 - \underbrace{p_H}_{\uparrow \text{ 2 failures }}) R_1 + (1 - p_H)^2 R_0$$

(conditional
on being
diligent)

Borrower incentive constraint:

Assume: $R_1 = R_0 = 0$

only positive reward
if both
projects
succeed

$$p_H^2 R_2 \geq p_L^2 R_2 + B$$

$$(p_H^2 - p_L^2) R_2 \geq B$$

$$(p_H - p_L)(p_H + p_L) R_2 \geq B$$

; Let's define $\Delta p \equiv p_H - p_L$.

$$R_2 \geq \frac{B}{\Delta p (p_H + p_L)}$$

Expected pledgeable income is therefore:

$$\underbrace{2p_H R}_{\text{expected cash flow}} - \underbrace{p_H^2 R_2}_{\substack{\text{expected payment to} \\ \text{borrower} \\ \text{for both} \\ \text{projects}}} = 2p_H R - \frac{2p_H^2 B}{(p_H + p_L) \Delta p} = \boxed{2p_H R - 2(1 - d_2) \frac{p_H B}{\Delta p}}$$

$$d_2 \equiv \frac{p_L}{p_L + p_H} \in (0, \frac{1}{2})$$

The project will be financed if expected pledgeable income exceeds investor outlay $2I - 2A$, so:

$$p_H \left[R - (1 - d_2) \frac{B}{\Delta p} \right] \geq I - A$$

investor outlay per project.

pledgeable income per project

The incentive constraint is looser than under project finance.

$$d_2 = \frac{p_L}{p_L + p_H}$$

$$p_L + p_H$$

$$\in (0, 1/2).$$

Benefit of cross pledging / diversification, captured by $d_2 > 0$!

Separate finance → looked the same, but with $d_2 = 0$.

Easier to simultaneously satisfy the lender break-even constraint, and the borrower incentive constraint under joint financing.



Endogenous correlation

If lenders break-even on average, then the borrower earns expected payoff

$$U_B = p_H^2 R_2 - 2A = 2p_H \underbrace{(R - I)}_0 > 0$$

That is, where:

$$R_2 = \frac{2p_H(R - I) + 2A}{p_H^2} > 0$$

Now suppose that after borrowing $2(I - A)$, and unbeknownst to the lender, the borrower can secretly substitute the two independent projects for two projects with correlation ρ , where the relevant probabilities are:

Both projects succeed: $p_H [1 - (1 - p_H)(1 - \rho)]$

Exactly one success: $2p_H(1 - p_H)(1 - \rho)$

Both projects fail: $(1 - p_H) [1 - p_H(1 - \rho)]$

$\rho = 0$	$\rho = 1$
p_H^2	p_H
$2p_H(1 - p_H)$	0
$(1 - p_H)^2$	p_L

Question

Suppose the borrower is free to choose whichever value of the correlation ρ than they want. Which value will the borrower choose? What is the intuition?

Discuss in pairs. Then go to socrative.com, room 897458, and write a short answer.

The answer can be of the form: " $\rho = \dots$ ", and 1 or 2 sentences of intuition.

$$\rho = 1$$

Maximizes the prob. of 2 projects succeeding, and hence the prob. of getting reward $R_2 > 0$.



Question

Suppose the borrower is free to choose whichever value of the correlation ρ than they want. Which value will the borrower choose? What is the intuition?

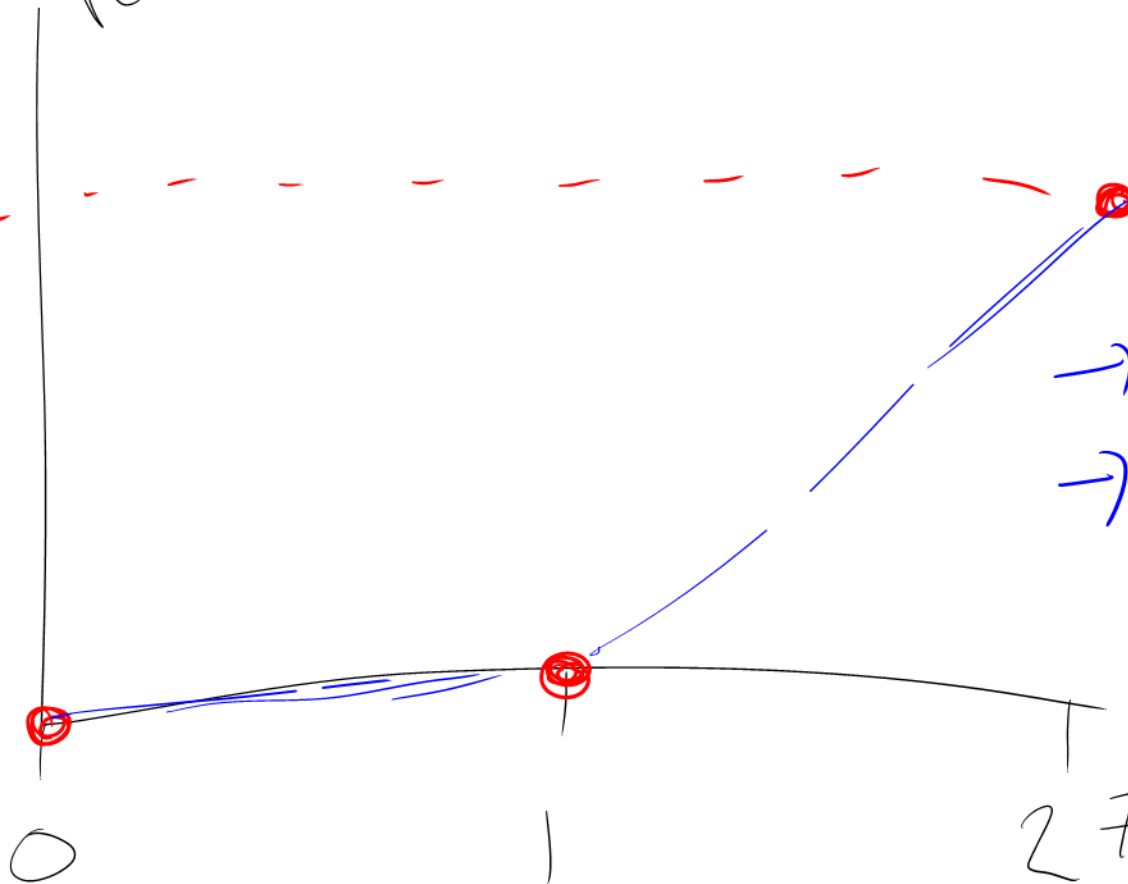
Which value will the borrower choose? Why?

Link to "risk-taking"
Ex ante, for incentives, best to have independent projects (easier to borrow).

Ex post, under the "high powered" contract based on independent projects, insider would prefer high correlation

Borrower
reward

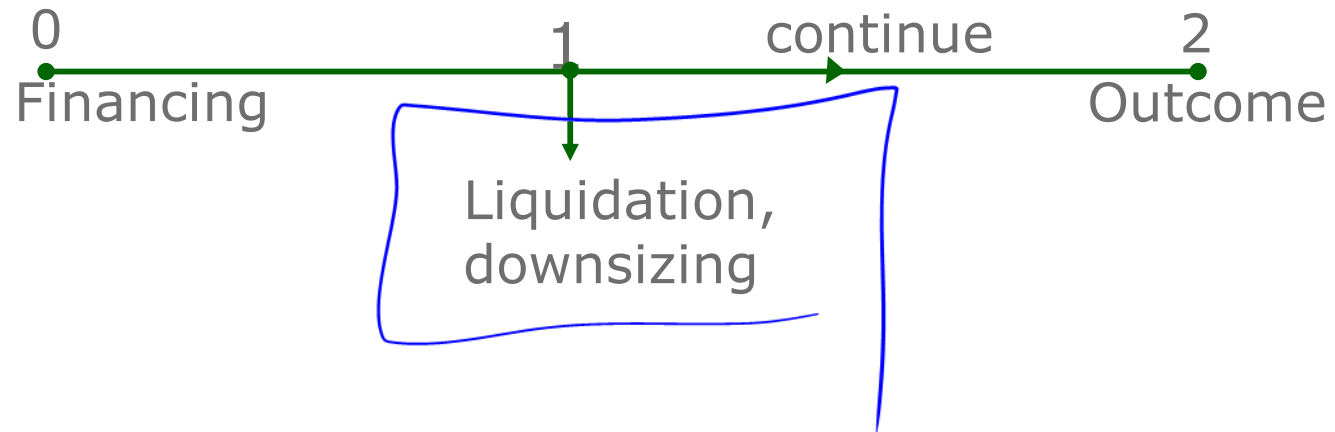
R_2



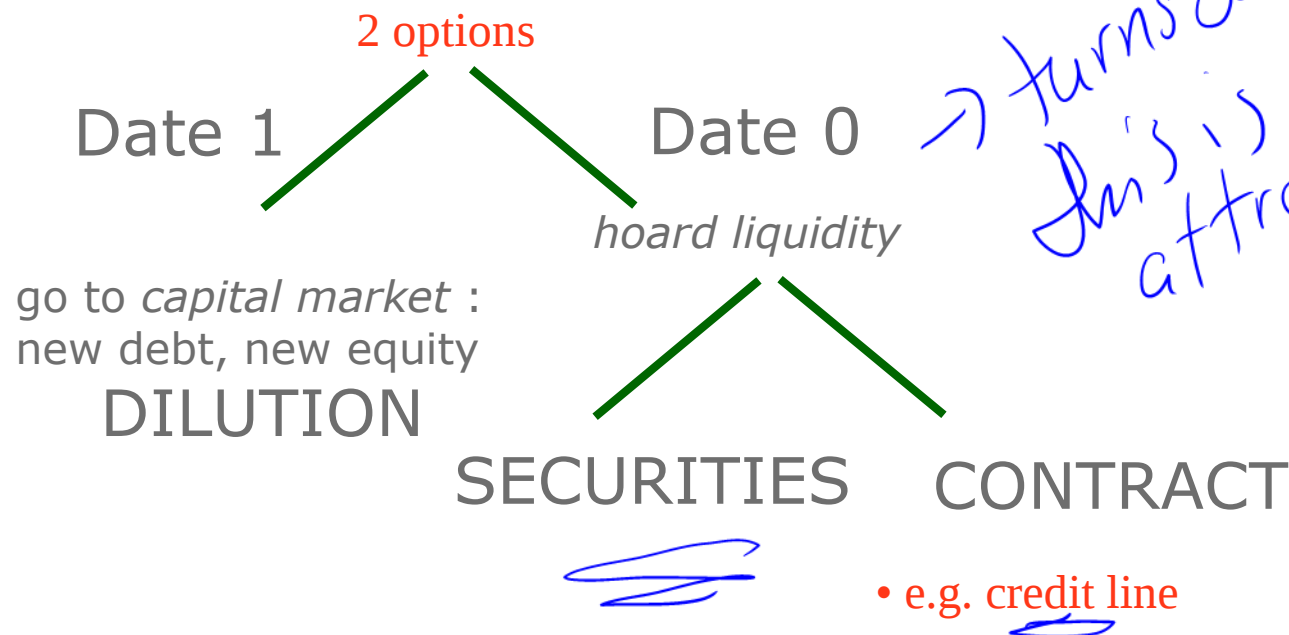
CORPORATE LIQUIDITY DEMAND

"Cash poor firm"

Cash need $<$ overruns/reinvestment
shortfall in earnings



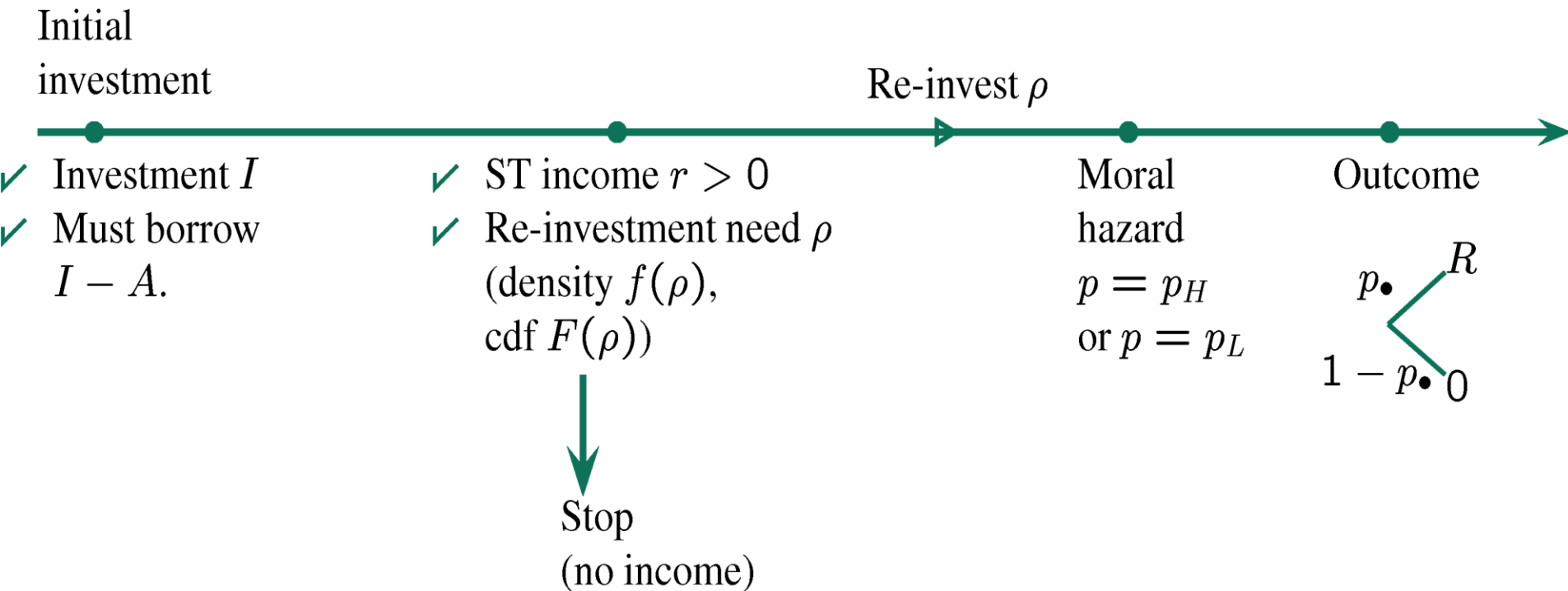
- How to meet these needs?



BASIC INSIGHT: LOGIC OF CREDIT RATIONING APPLIES AT DATE 1 AS WELL ⇒ WANT TO HOARD LIQUIDITY

CASH RICH FIRM: flip side of same coin

LIQUIDITY MANAGEMENT AND FREE CASH FLOW



Optimal policy: continue whenever $\rho \leq \rho^*$ for some ρ^* .



$$U_b(\rho^*) = \text{NPV}$$

$$= [r + F(\rho^*) p_H R] - \left[I + \int_0^{\rho^*} \rho f(\rho) d\rho \right]$$

$$(\text{IC}) \quad (\Delta p) R_b \geq B$$

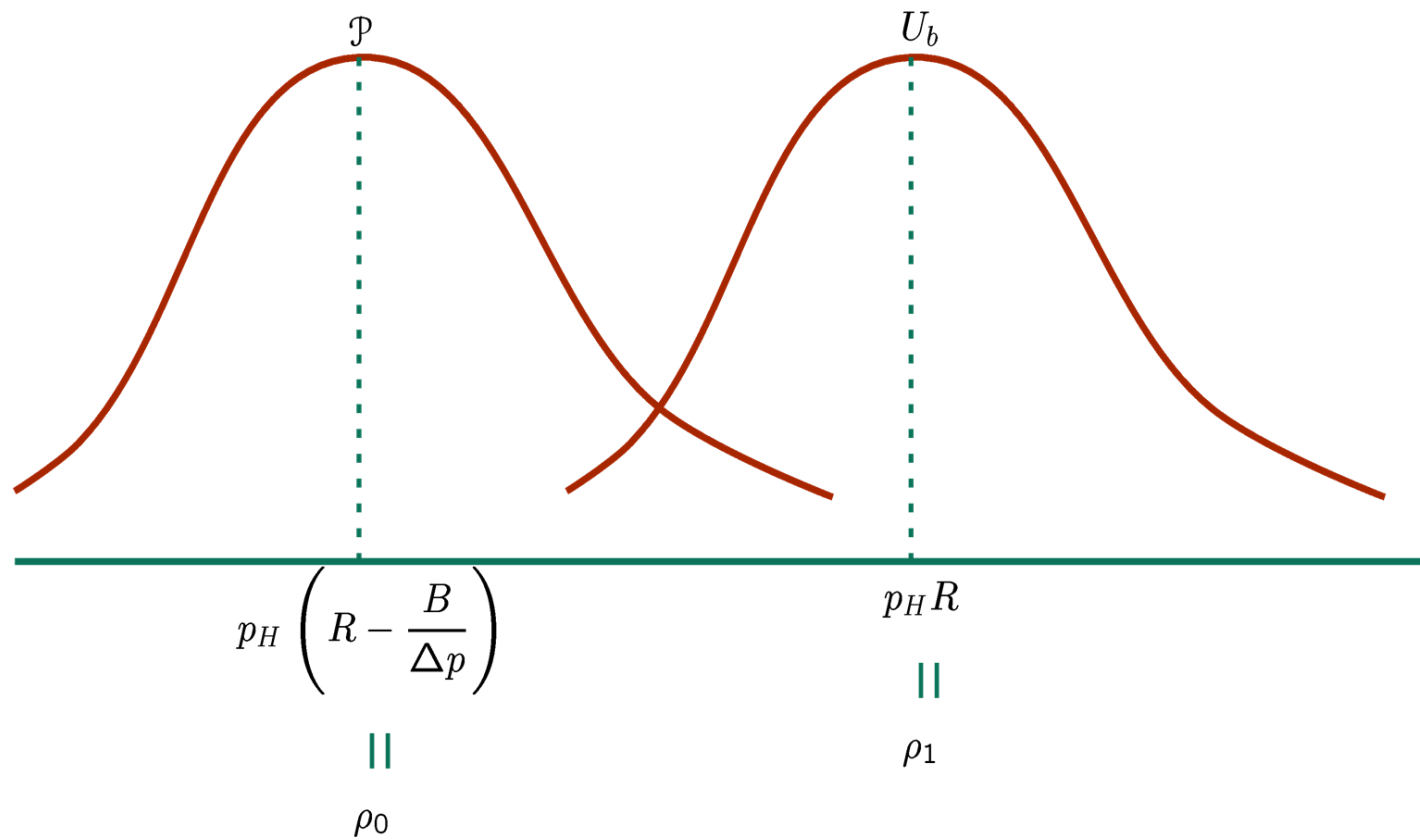
$$\begin{aligned} \mathcal{P}(\rho^*) - [I - A] &= \left[r + F(\rho^*) p_H \left(R - \frac{B}{\Delta p} \right) \right] \\ &\quad - \left[I + \int_0^{\rho^*} \rho f(\rho) d\rho \right] \end{aligned}$$



The NPV is increasing in the cutoff ρ^* as long as $\rho^* < p_H R$ and is decreasing thereafter.

Pledgeable income is increasing in the cutoff ρ^* as long as $\rho^* < p_H R(R - B/\Delta p)$ and is decreasing thereafter.





■ (i) $\mathcal{P}(p_H R) \geq I - A$

(first best)

$\Rightarrow \rho^* = p_H R$

■ (ii) $\mathcal{P}(p_H R) < I - A \leq \mathcal{P}\left(p_H \left(R \frac{B}{\Delta p}\right)\right)$

Then

$$\rho_0 < \rho^* < \rho_1$$

[Third case (iii)
no funding]

$\mathcal{P}\left(p_H \left(R - \frac{B}{\Delta p}\right)\right) < I - A \Rightarrow$



CASH-RICH FIRM: $r > \rho^$*

$$\text{ST debt} \quad d = r - \rho^*$$

$$\text{LT debt} \quad D = R - \frac{B}{\Delta p}$$

Theory of maturity structure:

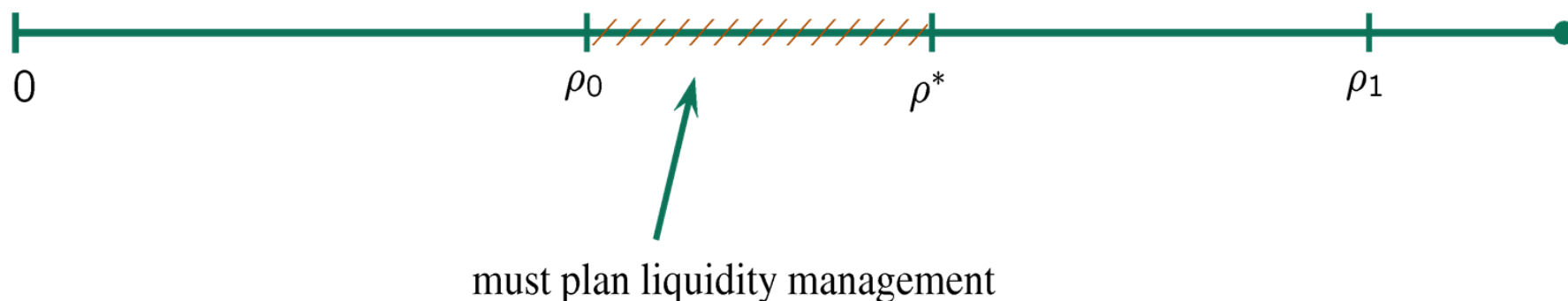
Weak balance sheet $(A \searrow) \Rightarrow \rho^* \searrow \Rightarrow d \nearrow$

$\Rightarrow$ *short maturity structure.*

CASH-POOR FIRM:

Example: $r = 0$.

"Wait-and-see" policy
suboptimal



Think of the lenders agreeing to a contract that allows them to break even on average.

The contract specifies that, if a liquidity shock hits, with $\rho < \rho^*$, then the firm borrows ρ from new lenders to meet the shock.

The firm promises repayment to new lenders in case of project success, so that these new lenders break even on average.



But what happens if: $\rho_0 < \rho < \rho_1$

i.e. $p_H(R - \frac{B}{\Delta p}) < \rho < p_H R$

The most the firm could possibly promise to repay new lenders in case of project success is $R - B/\Delta p$



Implication: the firm should “hoard liquidity” at date 0, before starting the project, rather than “wait and see” and try to borrow at date 1 if a liquidity shock hits.



Intended outcomes revisited

1. To describe how the potential for credit rationing can result in borrower's holding "inside equity", with lenders issuing "outside debt". A contract with inside equity and outside debt maximizes the borrower's incentive to behave by giving lenders a full claim on project returns in the case of failure; thus maximizing pledgeable income.
2. To derive how diversification (cross-pledging the returns of different projects) can affect credit rationing, and make the connection with asset substitution. Cross pledging can alleviate credit rationing by allowing a lender to receive repayment, even though the project they themselves funded failed. However, the borrower may be tempted to increase risk by choosing correlated projects (connection to asset substitution).
risk-taking
3. To extend the model of credit rationing in order to incorporate liquidity management, and show how its results relate to free-cash flow theory, debt overhang, and dilution. Here, liquidity management and free-cash theory are two sides of the same coin: preventing cash-poor firms from inefficiently liquidating / preventing cash-rich firms from inefficiently continuing. Debt overhang and dilution: see question for next time.
informally



Preparation for next time

1. Watch the video in the folder for Lecture 3 on collateral. Focus on understanding how asset redeployability affects the ability to borrow.
2. Consider the model of liquidity management and free cash flow seen today (slides 19 – 30) with a cash poor firm who is hit with a liquidity shock, and can either borrow from new lenders or scrap the project (in particular slides 28-30). But now assume that the liquidity shock comes as a **surprise** to all parties: initial lenders did not believe that a liquidity shock could occur, when they lent to the firm to start the project. Intuitively, describe whether initial lenders would accept that the firm issues debt to new lenders, in order to meet the shock? Think about the connection with our earlier discussion on debt overhang and dilution (Lecture 1 and video)

instead, look at the textbook chapter (see course plan on Absalom)

BE READY TO SHARE YOUR THOUGHTS NEXT TIME